ETH
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Theoretical Astrophysics and Cosmology

Exercise Sheet 2

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Exercise 1

Three supernovae are observed at redshifts $z = \{0.02, 0.2, 2.0\}$.

- Starting from the FLRW metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \left[\mathrm{d}\chi^2 + S_k^2(\chi)\mathrm{d}\Omega^2\right], \tag{1}$$

  and the luminosity distance

$$d_L = (1+z)a_0 S_k(\chi), \tag{2}$$

  show that the comoving distance χ can be computed as

$$\chi(z) = \int_0^z \frac{\mathrm{d}z'}{H(z')}, \tag{3}$$

  where you need to write down a suitable form for $H(z)$ to include the parameters of the cosmological model.

- Again starting from the metric, derive an expression for the elapsed cosmic time between explosion and detection of the supernova.

- Calculate the luminosity distance if the explosions takes place in a flat ($k = 0$) Universe with $\Omega_{\mathrm{m},0} = 0.3$, $\Omega_{\Lambda,0} = 0.7$, at $z = 0$. Take $H_0 = 72\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

  Also calculate the elapsed cosmic time between explosion and detection.

- As above, but for an open ($k < 0$) Universe with $\Omega_{\mathrm{m},0} = 0.3$ at $z = 0$. Don't forget the effective density from curvature!

  Also calculate the elapsed cosmic time between explosion and detection.

Exercise 2

Consider two comoving observers A and B, **each at the same comoving distance** χ away from you. At conformal time τ_e, they first come into causal contact: A photon emitted by A

at $\tau = 0$ reaches B at $\tau = \tau_e$ (and vice versa).
At $\tau = \tau_e$, both A and B emit photons toward you, which you receive today at $\tau = \tau_0$.
Assume you are in an Einstein-de Sitter Universe with $H_0 = 72\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

- Write down the FLRW metric from Eq. (1) in conformal time τ (instead of cosmic time t). Determine the scale factor as a function of cosmic time $a(\tau)$.

  Physically speaking, what is the difference between conformal and cosmic time?

- Determine the (i) comoving and (ii) proper distance between A and B as they enter into causal contact. Also calculate the (iii) comoving and (iv) proper distance at τ_e between A (B) and you.

  What do the comoving and proper distances tell you?

- What is the angular separation that you measure between A and B? Assume that the angle is small.

- Write the angular separation as a function of redshift z. What angular separation do you measure if A and B emit photons towards you at $z \sim 1067$?

  NB: This is the redshift corresponding to the surface of last scattering.

Exercise 3

The surface brightness (also known as intensity) of an astronomical object is given by

$$I = \frac{F}{\Omega}, \tag{4}$$

where Ω is the solid angle subtended by the source, and $F = L/A$ the flux emitted by the source of bolometric (integrated over all frequencies) luminosity L over an area A.

- How do luminosity and apparent angular distance scale with redshift? How does the surface brightness scale with redshift?

- Assume that an astronomical object is both a "standard ruler" and a "standard candle" – hence, you can measure both the luminosity distance and the angular diameter distance. Show that this kind of observation rules out the "tired light model".

  Hint: After astronomers such as Edwin Hubble have shown that distant objects appear more redshifted, Swiss astronomer Fritz Zwicky proposed the "tired light model" in 1929. In that model, photons would collide with other particles and (over cosmic time scales) loose energy. In such a model, how would the luminosity distance and the angular diameter distance scale with redshift?