ETH
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Theoretical Astrophysics and Cosmology

Exercise Sheet 4

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Exercise 1

In the very early Universe, free protons and free neutrons were in thermal equilibrium with the hot plasma. Their equilibrium was maintained by the two main reactions with electronic neutrinos and antineutrinos:

$$n + \nu_e \leftrightharpoons p + e^- \,, \qquad p + \bar{\nu}_e \leftrightharpoons n + e^+ \,. \tag{1}$$

Compute the value corresponding to the freeze-out temperature of the ratio between the number density of free neutrons and protons.

Exercise 2

1. **Hot relics:** The exact mass $\sum m_i$ of the neutrinos (summed over the three different flavors) is not known. Lab experiments can provide both lower and upper bounds on the mass. However, the best upper bounds are obtained from cosmological constraints.

   - Neutrinos have been relativistic when decoupling. Assuming that they are non-relativistic today, compute their relic contribution to the energy density $\rho_{\nu,0}(m_i)$ in dependence on the neutrino mass m_i.
   - Assuming that the neutrinos do not over-close the universe ($\rho_{\nu,0} \leq \rho_{\text{crit},0}$), show that an upper bound on the neutrino masses is given by

   $$\sum m_i h^2 \leq 93\,\text{eV}\,, \tag{2}$$

   where $H_0 = 100h\,\text{s}^{-1}\,\text{km/Mpc}$.

   Hint: What was the number density at decoupling, and how did it evolve afterwards? Does the relic abundance depend on the exact time T_{dec} of decoupling?

2. **Cold relics:** The lightest neutralino species predicted by supersymmetry should have a mass of at least 30 GeV, based on various experimental constraints provided by LHC.

   - What is its expected contribution to the density parameter of such a relic, and would it be consistent with cosmological constraints?
   - Explain how the decoupling time changes as the particle mass increases.

   Hint: Read pages 135–137 in "Galaxy Evolution and Formation" to answer this question.