ETH
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Theoretical Astrophysics and Cosmology

Exercise Sheet 5

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Exercise 1

After the freeze-out of the neutron-to-proton ratio, Big Bang nucleosynthesis starts. During this epoch, a small amount of tritium (denoted Tr or ^{3}H) is produced via the reactions

$$\mathrm{p} + \mathrm{n} \leftrightharpoons \mathrm{D} + \gamma\,, \qquad \mathrm{D} + \mathrm{n} \leftrightharpoons \mathrm{Tr} + \gamma\,, \qquad \mathrm{D} + \mathrm{D} \leftrightharpoons \mathrm{Tr} + \mathrm{p}\,. \tag{1}$$

Assuming that these reactions happen in thermal equilibrium, compute the abundance X_{Tr} of tritium as a function of temperature T, starting from the equilibrium number density of a non-relativistic species. Recall that the abundance X_A of a nucleus is defined as

$$X_A \equiv \frac{A n_A}{n_b}\,, \tag{2}$$

where n_b is the total number density of baryons in the Universe. Explain qualitatively: At which temperatures can a non-negligible amount of tritium be produced? Is the actual tritium abundance today the same as it was at the end of nucleosynthesis?

Exercise 2

When the temperature of the universe has cooled down to temperatures below $\sim 13.6\,\mathrm{eV}$, electrons and protons start to combine to form hydrogen atoms. However, direct recombination to the ground state of hydrogen does not decrease the free electron fraction, as the photon emitted in the process can immediately ionize another hydrogen atom. Therefore hydrogen recombination must occur through a two-step process: First electrons and protons recombine to the excited 2s hydrogen state,

$$\mathrm{e} + \mathrm{p} \rightleftharpoons \mathrm{H}_{2\mathrm{s}} + \gamma\,, \tag{3}$$

then this decays into the ground state 1s with the emission of two photons (two-photon decay). The rate of change in the free electron fraction x_e can be written as[1]

$$\frac{\mathrm{d}x_e}{\mathrm{d}t} = -\Gamma_{2\gamma} n_B \alpha x_e^2 (\Gamma_{2\gamma} + \beta)^{-1}\,, \tag{4}$$

where $\Gamma_{2\gamma} = 8.2\,\mathrm{s}^{-1}$ is the two-photon decay rate, n_B is the number density of neutral atoms and electrons, β is the ionization rate of the 2s level ($[\beta] = \mathrm{s}^{-1}$) and α is the hydrogen recombination coefficient ($[\alpha] = \mathrm{cm}^3\mathrm{s}^{-1}$).

[1] See Mukhanov, "Physical Foundations of Cosmology", page 126 for the derivation.

Verify that the two terms inside the paranthesis become comparable at a temperatur $T \approx 0.21\,\mathrm{eV}$, assuming that the reaction (3) takes place in thermal equilibrium such that

$$\frac{\mathrm{d}n_{H_{2s}}}{\mathrm{d}t} = \alpha n_p n_e - \beta n_{H_{2s}} = 0\,, \tag{5}$$

and the recombination coefficient α is well approximated by

$$\alpha(T) \approx 6.3 \cdot 10^{-14} \left(\frac{B_{H_{2s}}}{4T}\right)^{1/2} \mathrm{cm}^3\mathrm{s}^{-1}\,. \tag{6}$$

Explain qualitatively what happens to x_e when the universe cools down from temperatures well above $0.21\,\mathrm{eV}$ to temperatures well below $0.21\,\mathrm{eV}$.