ETH
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Theoretical Astrophysics and Cosmology

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Exercise Sheet 6

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Exercise 1

Helium recombination may be described as a two-step process: First, an α–particle captures the first electron. Then, a He^+ atom captures the second one to finally become He.

a) Consider the latter and assume that it occurs in equilibrium. Then, derive the expression for the ratio of the number density of He^+ and He and show that it is approximately unity at $T \simeq 6800$ K, and $\sim 10^{-4}$ at $T = 5600$ K.

b) Explain why the recombination temperature (when $\frac{n_{He^+}}{n_{He}} \leq 1$) is significantly smaller than the temperature corresponding to the ionization energy.

Hint: Use the fact that the ionisation energies of both He and He^+ are higher than the ionisation energy of hydrogen. Therefore they both recombine before hydrogen. Nevertheless, even complete recombination of helium only decreases the fraction of free electrons by 12% at most. Before hydrogen recombination, the number density of free electrons is thus given by

$$n_e(T) \simeq (0.75 \text{ to } 0.88) n_b(T) \simeq 0.2\eta T^3 \tag{1}$$

where n_b denotes the baryon density and η is the baryon-to-photon ratio. Furthermore, let us recall that the binding energy for the ground state of Helium is $B = 24.6\,\text{eV}$.

Exercise 2

Before decoupling, radiation and matter were in thermal equilibrium and the number density of photons (per frequency interval) follows the Planck distribution:

$$n_T(\nu)\mathrm{d}\nu = \frac{8\pi\nu^2}{c^3} \frac{\mathrm{d}\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1} \,. \tag{2}$$

a) The CMB maintains a black body spectrum from recombination until today, with temperature T that changes with time. Prove this behavior assuming that the decoupling occurs instantaneously at some t.

b) In reality decoupling is not instantaneous, but lasts for $\Delta z_{rec} \simeq 80$. Do photons that decoupled at $z_{rec} + \frac{\Delta z_{rec}}{2}$ produce the same black body spectrum as photons that decoupled at $z_{rec} - \frac{\Delta z_{rec}}{2}$?

Exercise 3

1. Write out the relation for the comoving distance travelled by a photon emitted at time t_e and received at time t_0. Use this relation to compute the size of the **particle horizon** with respect to the Hubble time for
   - A radiation dominated Universe,
   - A matter dominated Universe,
   - A vacuum energy (Λ) dominated Universe,
   - A curvature dominated Universe.
2. Now compute the size of the **event horizon** with respect to the Hubble time for each of the above Universes.
3. In light of the above results, consider the following. If the Universe is Λ (vacuum energy) dominated today, but contains a significant amount of matter, does this mean that a fundamental observer will eventually be able to be causally connected with all events in spacetime?