Gravitational Lensing

Theoretical Astrophysics and Cosmology

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What is weak gravitational lensing?

Strong gravitational lensing:

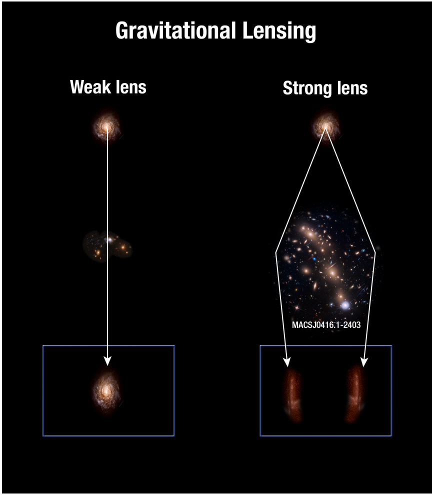
We see "weird" images.

Weak gravitational lensing:

Images are not weird at all.

Types of weak lensing:

- Weak lensing in galaxy clusters
- Cosmological weak lensing, affecting...
  - Galaxy shapes
  - CMB



Thin lens approximation: Geometric Setup

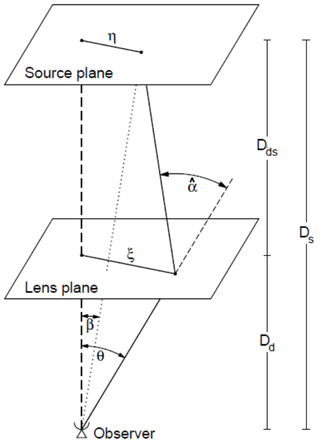


Image Credit: Bartelmann & Schneider (2001)

$\boldsymbol{\beta}\ldots$ source position
$\boldsymbol{\theta}\ldots$ observed source position
$\hat{\boldsymbol{\alpha}}\ldots$ deflection angle
$\boldsymbol{\alpha}\ldots$ scaled deflection angle

$$\boldsymbol{\theta} = \boldsymbol{\beta} + \frac{D_{ds}}{D_s}\hat{\boldsymbol{\alpha}} = \boldsymbol{\beta} + \boldsymbol{\alpha}$$

Notation lecture notes: $\xi \mapsto b$, $\boldsymbol{\beta} \mapsto \hat{\mathbf{s}}$, $\boldsymbol{\theta} \mapsto \hat{\mathbf{n}}$, $D_{ds} \mapsto D_{ls}$, $D_d \mapsto D_l$

Lensing by a point mass

Deflection angle: $\hat{\alpha} = \dfrac{4GM}{\xi c^2}$, Einstein radius: $\theta_E = \sqrt{\dfrac{4GM}{c^2}\dfrac{\mathcal{D}_{ds}}{\mathcal{D}_l \mathcal{D}_s}}$.

Note that this is valid for an impact parameter much larger than the Schwarzschild radius, $\xi \gg 2GM/c^2$. Hence, the deflection angle $\hat{\alpha}$ is small.

Image credit: ESA/Hubble & Nasa

Lensing by a 2D mass distribution

The (scaled) deflection angle $\boldsymbol{\alpha}(\boldsymbol{\theta})$ is determined by the *lensing potential* $\boldsymbol{\Phi}$:

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \nabla\boldsymbol{\Phi}(\boldsymbol{\theta})\,, \qquad \boldsymbol{\Phi}(\boldsymbol{\theta}) = \frac{1}{\pi}\int \mathrm{d}^2\boldsymbol{\theta}'\,\kappa(\boldsymbol{\theta}')\ln|\boldsymbol{\theta}-\boldsymbol{\theta}'|\,.$$

Dimensionless surface mass density / convergence $\kappa(\boldsymbol{\theta})$:

$$\kappa(\boldsymbol{\theta}) := \frac{\Sigma(D_d\boldsymbol{\theta})}{\Sigma_{cr}}\,, \qquad \Sigma_{cr} = \frac{c^2}{4\pi G}\frac{D_s}{D_d D_{ds}}\,.$$

Critical surface mass density Σ_{cr}:

$$\begin{array}{lll} \kappa \geq 1\,, & \Sigma \geq \Sigma_{cr}: & \text{Multiple images} \Rightarrow \text{Strong lensing} \\ \kappa < 1\,, & \Sigma < \Sigma_{cr}: & \text{Weak lensing} \end{array}$$

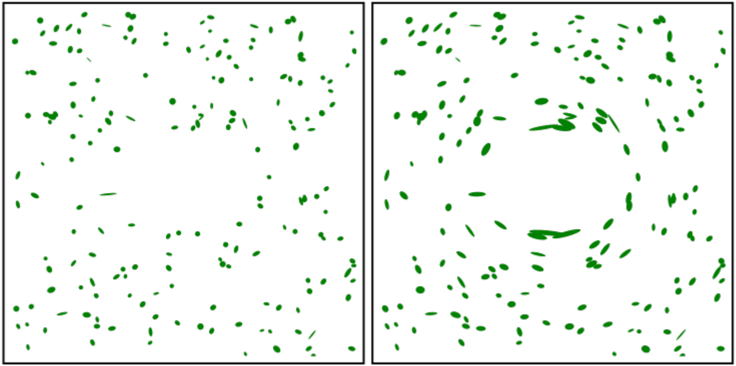
Strong lensing: Can be used for mass reconstruction.

Are Weak Lensing Effects useless?

“You have a mass distribution about which you don’t know anything, and then you observe sources which you don’t know either, and then you claim to know something about the mass distribution?”

Some guy from some PhD examination committee,
according to P. Schneider.

Weak lensing effects are statistical effects!



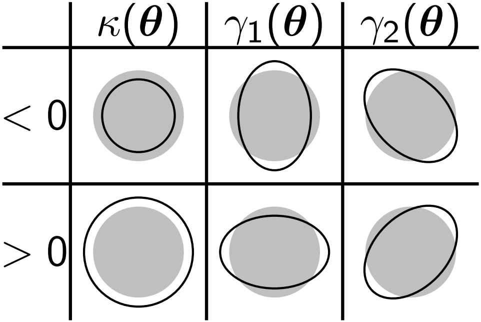
Averaging over many background galaxy shapes, we can observe common patterns!

Effects of γ and κ on Image Shapes

The distortion of images is described by the Jacobian matrix:

$$\mathcal{A}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} = \left(\delta_{ij} - \frac{\partial^2 \boldsymbol{\Phi}(\boldsymbol{\theta})}{\partial \theta_i \partial \theta_j} \right) = \begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix} .$$

$\gamma = \gamma_1 + \mathrm{i}\gamma_2 \ldots$ *shear* components.



Everything is determined by $\kappa(\theta)$!

Poisson's equation:

$$2\kappa(\boldsymbol{\theta}) = \nabla^2 \Phi(\boldsymbol{\theta})\,.$$

The shear is related to the surface density as:

$$\gamma(\boldsymbol{\theta}) = \frac{1}{\pi}\int \mathrm{d}^2\boldsymbol{\theta}' \mathcal{D}(\boldsymbol{\theta}-\boldsymbol{\theta}')\kappa(\boldsymbol{\theta}')\,, \qquad \mathcal{D}(\boldsymbol{\theta}) := \frac{-1}{(\theta_1 - \mathrm{i}\theta_2)^2}\,.$$

This relation can be inverted:

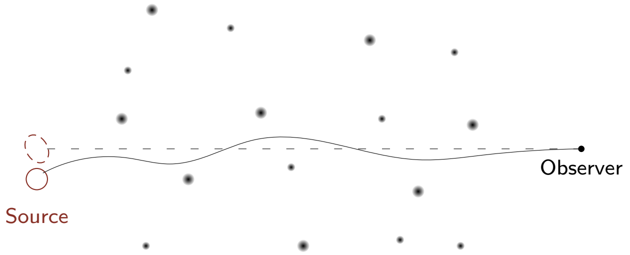
$$\kappa(\boldsymbol{\theta}) = \frac{1}{\pi}\int \mathrm{d}^2\boldsymbol{\theta}' \mathcal{D}^*(\boldsymbol{\theta}-\boldsymbol{\theta}')\gamma(\boldsymbol{\theta}')\,.$$

Strong lensing: Mass distribution in the inner part of the cluster.

Weak lensing: Mass distribution at larger separations from the cluster center.

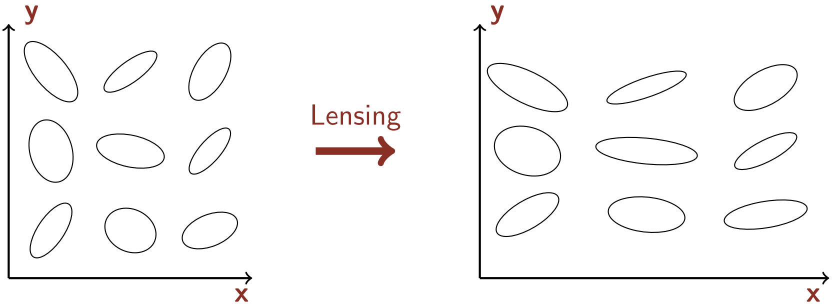
Cosmological Weak Lensing

= Weak gravitational lensing by large scale structures.



Thin lens approximation is not valid anymore!

To detect a cosmological weak lensing signal, we need to average over many galaxy shapes.



Cosmic shear: Common alignment of the galaxies.

Cosmological Weak Lensing Formalism

Consider a set of sources (galaxies) located at a radial distance r_s. We can express the deflection angle $\boldsymbol{\alpha}(\boldsymbol{\theta})$ in terms of the *deflection potential* $\boldsymbol{\Phi}(\boldsymbol{\theta})$:

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \nabla \boldsymbol{\Phi}(\boldsymbol{\theta}) , \qquad \boldsymbol{\Phi}(\boldsymbol{\theta}) = \int_0^{r_s} \mathrm{d}r \left(\frac{r_s - r}{r_s r} \right) 2\psi(r, \boldsymbol{\theta}) .$$

$\psi(r, \boldsymbol{\theta}) \ldots$ gravitational potential along the light path.

The distortion of images is described by the Jacobian matrix:

$$\mathcal{A}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} = \left(\delta_{ij} - \frac{\partial^2 \boldsymbol{\Phi}}{\partial \theta_i \partial \theta_j} \right) = \begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix} - \begin{pmatrix} \gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1 \end{pmatrix} .$$

$\gamma = \gamma_1 + \mathrm{i}\gamma_2 \ldots$ *cosmic shear* components, $\quad \kappa \ldots$ *convergence*.

Probing the Matter Distribution of the Universe

The cosmological weak lensing observables are determined by the *density perturbations* δ_m of the universe:

$$\Delta\psi(r,\boldsymbol{\theta}) = \frac{3H_0^2\Omega_m}{2a}\delta_m(r,\boldsymbol{\theta})\,, \qquad 2\kappa(\boldsymbol{\theta}) = \Delta\Phi(\boldsymbol{\theta})\,.$$

Relation between κ and δ_m:

$$\kappa(\boldsymbol{\theta}) = \frac{3H_0^2\Omega_m}{2}\int_0^{r_s}\frac{\mathrm{d}r}{a(r)}\frac{(r_s-r)r}{r_s}\delta_m(r,\boldsymbol{\theta})\,.$$

Cosmological weak lensing effects are determined by the density perturbations δ_m of the Universe.

Shear Patterns

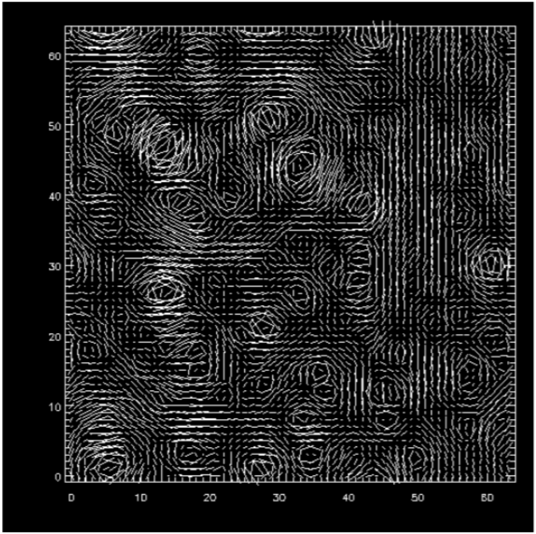


Image credit: A. Refregier (2003)

Cosmic Shear as a Cosmological Probe

Measuring cosmic shear patterns gives us information about the matter distribution!

We gain information on the matter power spectrum $P_m(k)$!
Depends on cosmological parameters (Ω_m, Ω_Λ, Ω_r, h, σ_8).

Cosmic shear measurement: First achieved in 2000.

More reasons for why weak lensing is useful:

- It probes the matter distribution at late times, $z \leq 1$.
- It is directly related to the (dark) matter distribution, no further assumptions are needed.
- It can be combined with other cosmological probes to reduce parameter degeneracies.

Combining Weak Lensing with other Cosmological Probes

$\Rightarrow$ More sensitive to different combinations of σ_8 and Ω_m.

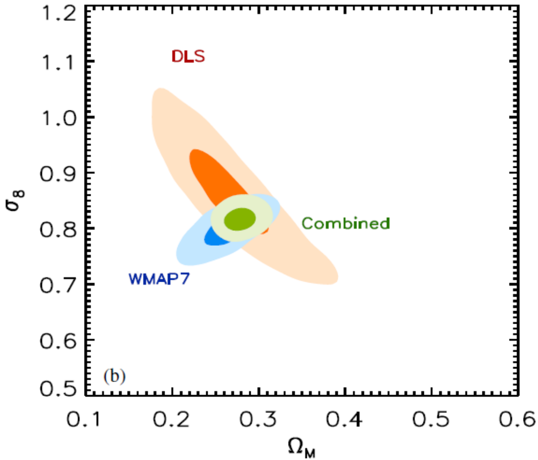
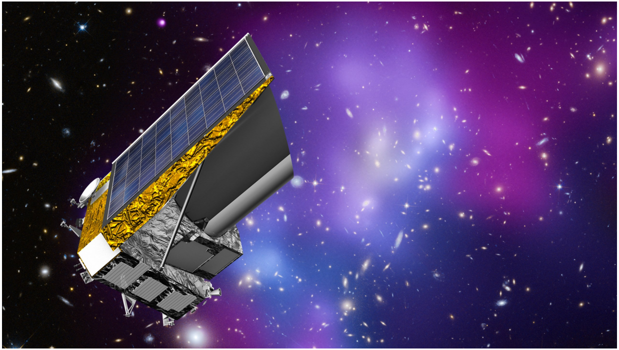


Image credit: Jee et al. (2013)

Future Cosmic Shear Observations

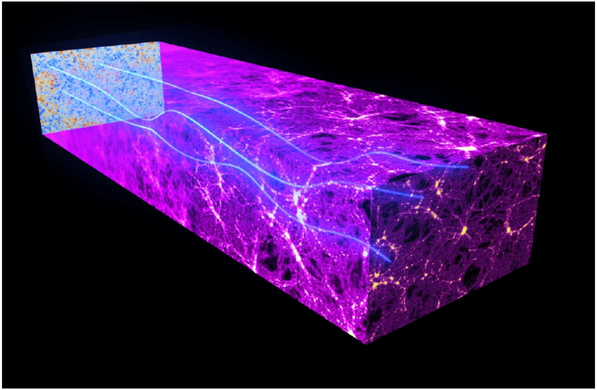
Planned surveys: LSST, WFIRST, Euclid.



These surveys will measure roughly a billion galaxies a large fraction of the sky (15,000 deg^2) with high precision.

Image: Artist's impression of Euclid; Copyright ESA/C. Carreau

Cosmological Weak Lensing of the CMB

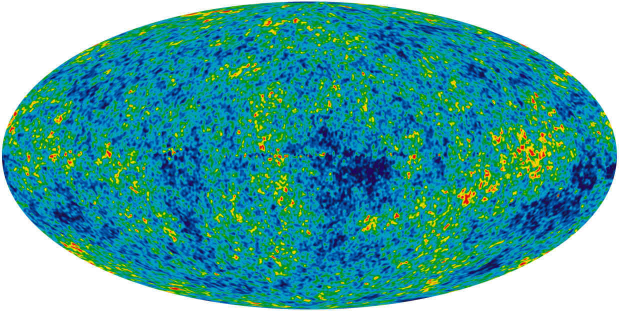


Matter affects the path of ALL photons in the universe!

Image credit: ESA

CMB: Basic properties

- CMB photons are freely streaming since $z \sim 1050$.
- CMB has an almost uniform temperature $T = 2.725\,\mathrm{K}$.
- Temperature fluctuations of $\sim 10^{-4}\,\mathrm{K}$ ("CMB anisotropies").



The statistics of these anisotropies depend on our cosmological model!

Image credit: NASA / WMAP Science Team

Effect on Temperature Anisotropies

- Cosmological weak lensing affects the statistical properties of the CMB.
- In particular, it induces statistical properties which are not expected for the unlensed CMB.
- We can extract the lensing signal from our CMB observations!

From observing the (lensed) CMB anisotropies, we obtain two distinct cosmological probes, the unlensed CMB anisotropies and the CMB lensing.

Both are useful to constrain cosmological parameters.

Summary

- Strong lensing: Multiple images, weirdly deformed images (extreme case: Einstein ring).
- Weak lensing effects are purely statistical!
- In galaxy clusters, both strong lensing and weak lensing can occur and can be used for mass reconstruction.
- Cosmological Weak lensing: Powerful cosmological probe, both in the case of cosmic shear (affects galaxy shapes) and in the case of CMB lensing.