Matter Power Spectrum (today)

* matter (pressureless)

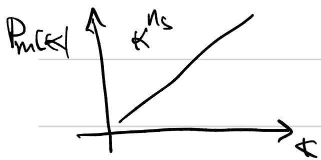
evolution of matter δ, $\delta(x,t) = D(t)\,\delta_i(x)$, $\ddot{D} + 2H\dot{D} - 4\pi G\bar{\rho}_m D = 0$

two-point correlation $\xi = \langle \delta(x)\,\delta(x+r) \rangle \xrightarrow{FT}$ power spectrum $P(k,t)$

initial condition : Gaussian random fluctuation. $\delta(r)_i$ $\sigma^2(k) \equiv P(k)$ at initial time

scale-invariant $\Delta^2_\phi \equiv \frac{k^3 P_\phi(k)}{2\pi^2} \sim A_s\left(\frac{k}{k_0}\right)^{n_s - 1}$, $n_s \simeq 1$ $P_\phi(k) \sim k^{n_s-4}$

3D $\delta(\vec{x})$, 2D $\delta(\hat{x}_0)$ or $\delta(\hat{n})$, 1D $\delta(z)$ $\delta_k \sim -k^2\phi_k \leftrightarrow \delta \sim \Delta\phi$ $P_m \sim k^{n_s}$



RDE, superhorizon scale : pressure, GR description

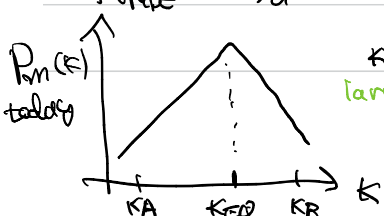
Newtonian description : OK on large scales in RDE $k < H$

RDE $H^2 \sim \rho_{RDE} \sim \frac{1}{a^4}$, $H \sim \frac{1}{a^2}$. $D_{RDE} \sim a^2$

small scales in RDE : radiation pressure prevents any growth for no growth $k > H$

RDE → MDE Equality $\rho_{rad}(a_{EQ}) = \rho_m(a_{EQ})$ a_{EQ}. $H_{EQ} = k_{EQ}$

$H^2_{MDE} \sim 1/a^3$, $H \sim a^{-3/2}$, $D_{MDE} \sim a$ scale-independent



$k_A < k_{EQ} < k_B$

large EQ small

$\delta(k_A, t_0) = \delta_i(k_A)\left(\frac{a_{EQ}}{a_i}\right)^2\left(\frac{a_0}{a_{EQ}}\right)$

$\delta(k_B, t_0) = \delta_i(k_B)\left(\frac{a_*}{a_i}\right)^2\left(\frac{a_0}{a_{EQ}}\right)$

$k_B = H_{RDE}(a_*)$

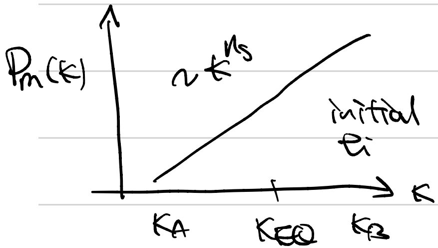
$H_0 = 100h\,\mathrm{km/s/Mpc} \sim \frac{1}{T} \sim \frac{1}{L} \quad (c=1)$

RDE

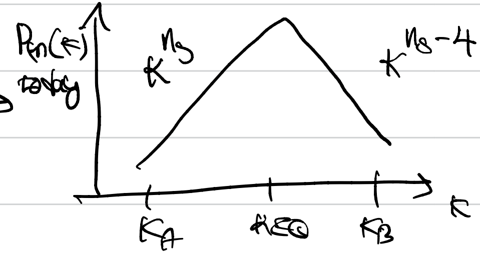
$k_B \sim \mathcal{H}(a_*)$. $\quad H^2_{RDE} \sim \frac{1}{a^4}$, $\quad H_{RDE} \sim \frac{1}{a^2}$ $\quad H = \frac{\dot a}{a} = \frac{d\ln a}{dt} \sim \frac{1}{a^2} \rightarrow a\,da \sim dt = a\,d\eta \rightarrow da \sim d\eta$

$a_{RDE} \sim \eta$, $\quad \mathcal{H} \equiv aH \sim \frac{1}{a} \sim \frac{1}{\eta}$ $\quad k_B \sim \mathcal{H}(a_*) \sim \frac{1}{\eta_*} \sim \frac{1}{a_*}$

$$\frac{\delta(k_B, t_0)}{\delta(k_A, t_0)} = \frac{\delta_i(k_B)}{\delta_i(k_A)}\left(\frac{a_*}{a_{EQ}}\right)^2 \qquad \frac{P(k_B, t_0)}{P(k_A, t_0)} = \left(\frac{P_i(k_B)}{P_i(k_A)}\right)\left(\frac{a_*}{a_{EQ}}\right)^4 \sim \left(\frac{1}{k_B}\right)^4$$



$\xrightarrow{\quad}$ RDE MDE



Caveats

- linear calculation
- super horizon scales → GR
- matter : dark

Hubble expansion in RDE is fast $\qquad$ grow in RDE $\sim \ln k$

$\longleftarrow$ inflaton, ϕ $\qquad$ dark matter, baryon, H, He, Li, … γ, ν …

inflation $\quad \phi_k \rightarrow$ inflaton decay $\rightarrow$ RDE (BBN, …) $\longrightarrow$ MDE : gravitational instability

pressure : strong, no significant matter growth $\qquad$ stars, galaxies, planets, life, observations today, $P_m(k)$

galaxies

$\delta_m(\vec x)$ 3D
$\delta_m(\hat n)$ 2D $\rightarrow \delta_g \sim \delta_m$
$\delta_m(z)$ 1D

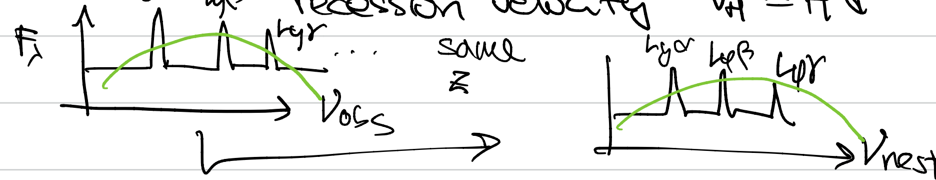
Peculiar velocity

δm, v, $\theta \equiv -\frac{1}{a}\nabla\cdot\vec{v}$

redshift z $\qquad 1+z \equiv \frac{\lambda_{obs}}{\lambda_{src}}$

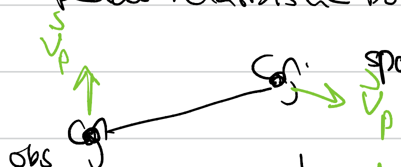
$1+z \simeq 1+v/c$, $\quad v = cz$

recession velocity $\quad v_H = Hd$



Doppler effect vs. Expansion of the Universe. can $z>1$? $\rightarrow$ $v>c$

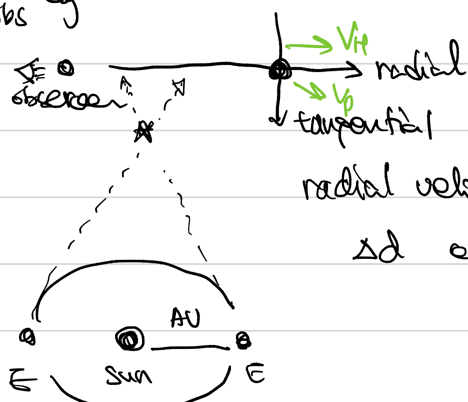
Special relativistic Doppler effect $\frac{\Delta\lambda}{\lambda_{src}}$ $\rightarrow$ $v \le c$ $\quad$ not correct interpretation



space is expanding

$\vec{V} = \vec{V}_H + \vec{V}_p \qquad \vec{V}_H$: radial

radial $\qquad$ radial & tangential



1 arcsec $\rightarrow$ pc $\qquad$ $\Delta\theta$ in angle $\Rightarrow$ Δd tangential

radial velocity at $d = 50$ Mpc/h $\qquad V_H = 100 \times 50$ km/s $= 5000$ km/s

Δd error in d $\simeq 10\%$ $\rightarrow$ $\Delta V_H \simeq 500$ km/s

$\vec{v} \equiv -\vec{\nabla} u + \cancel{\vec{v}^{(v)}}$, $\vec{v}^{(v)} \sim 1/a$ $\vec{v} = -\vec{\nabla} u$, $\theta = -\frac{1}{a}\vec{\nabla}\cdot\vec{v} = \frac{1}{a}\Delta u$

linear order solution $\theta = Hf\delta_m$ $f = \frac{d\ln D}{d\ln a}$ logarithmic growth rate

$\delta_m = D(t)\,\delta_i$ $\vec{v}$ or u or $\theta \propto HfD$ sensitive to gravity

other gravity model $D(t)$ at t_0 . derivative $\rightarrow f$

How to measure $\vec{v}$? $\vec{v}(\vec{x})$ 3D $\rightarrow \vec{v}_k \rightarrow P_{\vec{v}}(\vec{k})$

$\vec{v}(\vec{n})$ 2D $v_\parallel$ radial velocity

$\vec{v}(z)$ 1D $v_\perp$ tangential

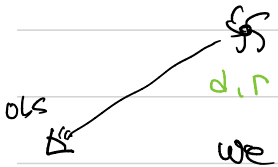
$\rightarrow$ two-correlation $\langle \vec{v}(x)\,\vec{v}(x+r)\rangle$

$$F(\bar{x}+\Delta) = F(\bar{x}) + \frac{dF}{dx}\Big|_{\bar{x}} \cdot \Delta + \cdots$$

$$s = \int^{z} \frac{dz}{H} = \int^{\bar{z}} \frac{dz}{H} + \frac{1}{H}(1+\bar{z})\delta z + \cdots$$

Redshift-space distortion

vs real-space



$z^{obs} = \bar{z} + \Delta$ $\qquad$ $z^{obs} \neq \bar{z}$ $\quad$ redshift-space vs real-space

$v^{obs} = v_H + v_p$ $\qquad$ impact : shifting the position in redshif-space

we need two-point $\xi(r)$ or $P(k)$ $\qquad$ Q: impact on $P(k)$

* real-space quantity : $\bar{z}$, r, ξ, $P(k)$ $\qquad 1+z = 1/a \qquad a = \frac{1}{1+z}$

$$ds^2 = 0 \rightarrow a^2 dr^2 = dt^2 = a^2 d\eta^2 \rightarrow dr = d\eta = \frac{d\eta}{da}\frac{da}{dz}dz = \frac{1}{a'}\frac{-1}{(1+z)^2}dz = -\frac{a^2}{a'}dz = -\frac{dz}{H}$$

$$r = \int_0^{\bar{z}} \frac{dz}{H}, \qquad d = a \cdot r = \frac{r}{1+\bar{z}}$$

* redshift-space quantity : z, s, ξ_z, $P_z(k)$

$$1+z \equiv (1+\bar{z})(1+\delta z), \qquad z = \bar{z} + (1+\bar{z})\delta z, \qquad \delta z \simeq v_p + \cdots$$

$$s = \int_0^{z} \frac{dz}{H} = \int^{\bar{z}} \frac{dz}{H} + \frac{1+\bar{z}}{H}\delta z + O(2) = r + \frac{v_p}{H} + \cdots \qquad s = r + v$$

$$\vec{v} = -\vec{\nabla} U, \qquad \theta = -\frac{1}{a}\vec{\nabla}\cdot\vec{v} = \frac{1}{a}\Delta U \overset{FT}{\longleftrightarrow} \theta_k = -\frac{k^2}{a}U_k = Hf\,\delta_k$$

$$= Hf\,\delta_m$$

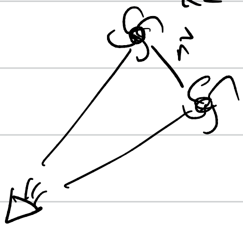
$$U = \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot x} U_k \qquad\qquad \therefore\ U_k = -\frac{Hf\,\delta_k}{k^2}$$

$$\vec{\nabla} U = \int \frac{d^3k}{(2\pi)^3} e^{ik\cdot x}\, i\vec{k}\, U_k \qquad\qquad v_k = i\vec{k}\,\frac{Hf\,\delta_k}{k^2}$$

$\mu_k = \hat{k}\cdot\hat{z} = \cos\theta_{k\cdot z}$

$k\mu_k$

$V_k = i\vec{k}\,\frac{\mathcal{H} f \delta_k}{k^2}$ $\qquad V_p = \hat{r}\cdot\vec{V} \rightarrow V_p(k) = i\vec{k}\cdot\hat{z}\,\frac{\mathcal{H} f \delta_k}{k^2} = i\,\frac{\mathcal{H} f \mu_k}{k}\,\delta_k$



$\langle \delta_g(\vec{x})\,\delta_g(\vec{x}+\vec{r})\rangle_{\vec{x}} \equiv \xi(r) \xleftrightarrow{FT} P(k)$

theory $\qquad$ obs $\qquad$ want to know

$n_g(\vec{x}) = \bar{n}_g(\bar{z})\,(1+\delta_g(x))$ $\qquad n_g(\vec{s}) = \bar{n}_g(z)\,(1+\delta_z(s))$

total # of objects in a given volume $\qquad n_g(x)\,d^3r = n_g(s)\,d^3s$

$1+\delta_z(s) = \frac{\bar{n}_g(\bar{z})\,r^2}{\bar{n}_g(s)\,s^2}\frac{dr}{ds}\,(1+\delta_g(x))$

no angular distortion

$d^3r = r^2\,dr\,d\Omega_r$

$d^3s = s^2\,ds\,d\Omega_s$

lensing effect $d\Omega_r \neq d\Omega_s$ ignored

$\bar{n}_g(s)\,s^2 = \bar{n}_g(r)\,r^2 + \frac{d}{dr}\left(\bar{n}_g(r)\,r^2\right)\cdot\nu + \cdots$

$s = r + v_p/\mathcal{H} \equiv r + \nu$

$\alpha = \frac{d\ln(n_g r^2)}{d\ln r}$

$\frac{n_g(s)\,s^2}{n_g(r)\,r^2} = 1 + \frac{d\ln(\bar{n}_g r^2)}{d\ln r}\,\frac{\nu}{r} + \cdots = 1 + \frac{\alpha}{r}\nu$

$1+\delta_z = \left(1+\frac{\alpha}{r}\nu\right)^{-1}\left(1+\frac{d\nu}{dr}\right)^{-1}(1+\delta_g) = 1 - \frac{\alpha}{r}\nu - \frac{d\nu}{dr} + \delta_g$

$\therefore\ \delta_z = \delta_g - \frac{d\nu}{dr} - \frac{\alpha}{r}\nu \simeq \delta_g - \frac{d\nu}{dr}$

$\mu_k = \cos(\hat{k}\cdot\hat{z})$

$v_p(k) = \frac{i aHf\mu_k}{k}\delta_k$ radial

$\nu(k) = \frac{v_p(k)}{\mathcal{H}} = \frac{i f \mu_k}{k}\delta_k$

$\frac{d}{dr} = \hat{n}\cdot\vec{\nabla} \to i k \mu_k \times$

$\frac{d\nu}{dr} = - f\mu_k^2 \delta_k$

RSD (redshift-space distortion)

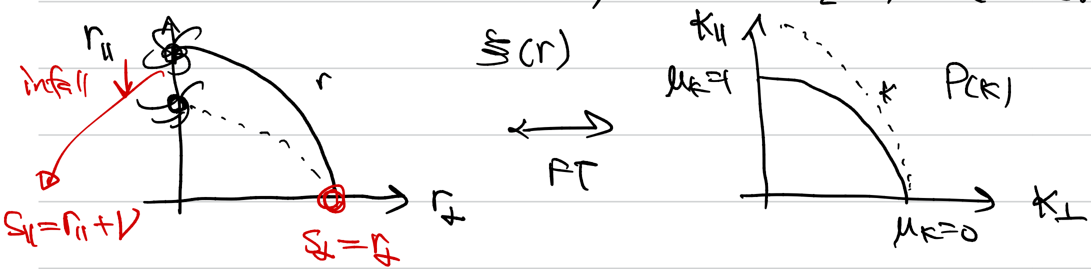
$\delta_z = \delta_g - \frac{d\nu}{dr}$, $\nu = \frac{v_p}{\mathcal{H}}$

$\Rightarrow \delta_g(k) + f\mu_k^2\,\delta(k)$

galaxy matter

galaxy & matter : linear bias relation $\delta_g(x) = b\cdot\delta_m(x) \to \delta_g(k) = b\cdot\delta(k)$

$\delta_z(k) = (b + f\mu_k^2)\,\delta_m(k)$ $P_z(k) = (b + f\mu_k^2)^2\,P_m(k)$

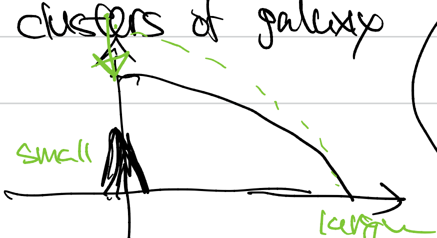


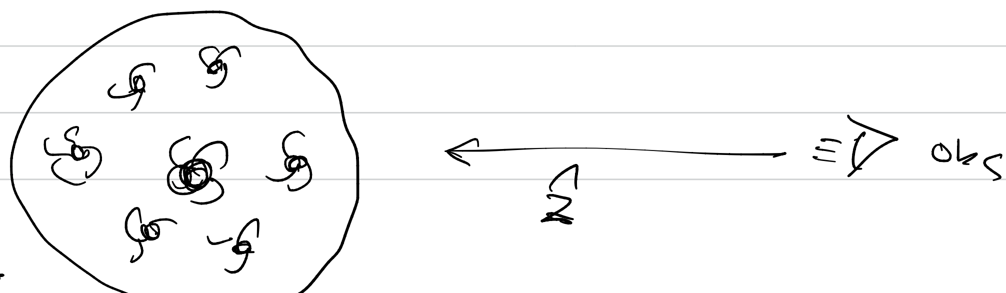
$k_\parallel = k\mu_k$

so far linear theory

small scales

clusters of galaxy





2pt − statistics $\langle \delta_k \rangle = \mu_k = 0$, $\langle \delta_k \delta_k \rangle \sim P(k)$

1pt − statistics $\rho_m(x) = \bar{\rho}_m(z)(1+\delta_m)$ $\langle \delta_m \rangle = 0$, $\bar{\rho}_m(z) \neq 0$

clusters of galaxies $n_c(x) = \bar{n}_c(z)(1+\delta_c)$ $\langle \delta_c \rangle = 0$, $\bar{n}_c(z) \neq 0$

Relativistic Perturbation Theory

* Governing eq : general relativity, fluid equation , PT

* Robertson-Walker : $ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -a^2 d\eta^2 + a^2 \bar{g}_{\alpha\beta} dx^\alpha dx^\beta$

μ, ν : space-time index 0, 1, 2, 3 , α, β : space index 1, 2, 3

$\bar{g}_{\alpha\beta}$: background 3-metric (or space metric) $\neq$ spatial part $g_{\mu\nu}$ in background.

$K=0$ $\bar{g}_{\alpha\beta} = \delta_{\alpha\beta}$ in rectangular coordinate vs $a^2 \delta_{\alpha\beta}$

* real univ. inhomogeneous univ $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

$g_{00} \equiv -a^2 (1 + 2\underset{\uparrow 1}{A})$, $g_{0\alpha} \equiv -a^2 \underset{\uparrow 3}{B_\alpha}$, $g_{\alpha\beta} = a^2 (\bar{g}_{\alpha\beta} + 2 \underset{\uparrow 6}{C_{\alpha\beta}})$

$g_{\mu\nu}$: symmetric

→ 10 dof $A = A^{(1)} + \cancel{A^{(2)}} + \cancel{A^{(3)}} + \cdots$

inverse metric tensor $g^{\mu\nu}$. $g^{\mu\rho} g_{\rho\nu} = \delta^\mu_\nu$