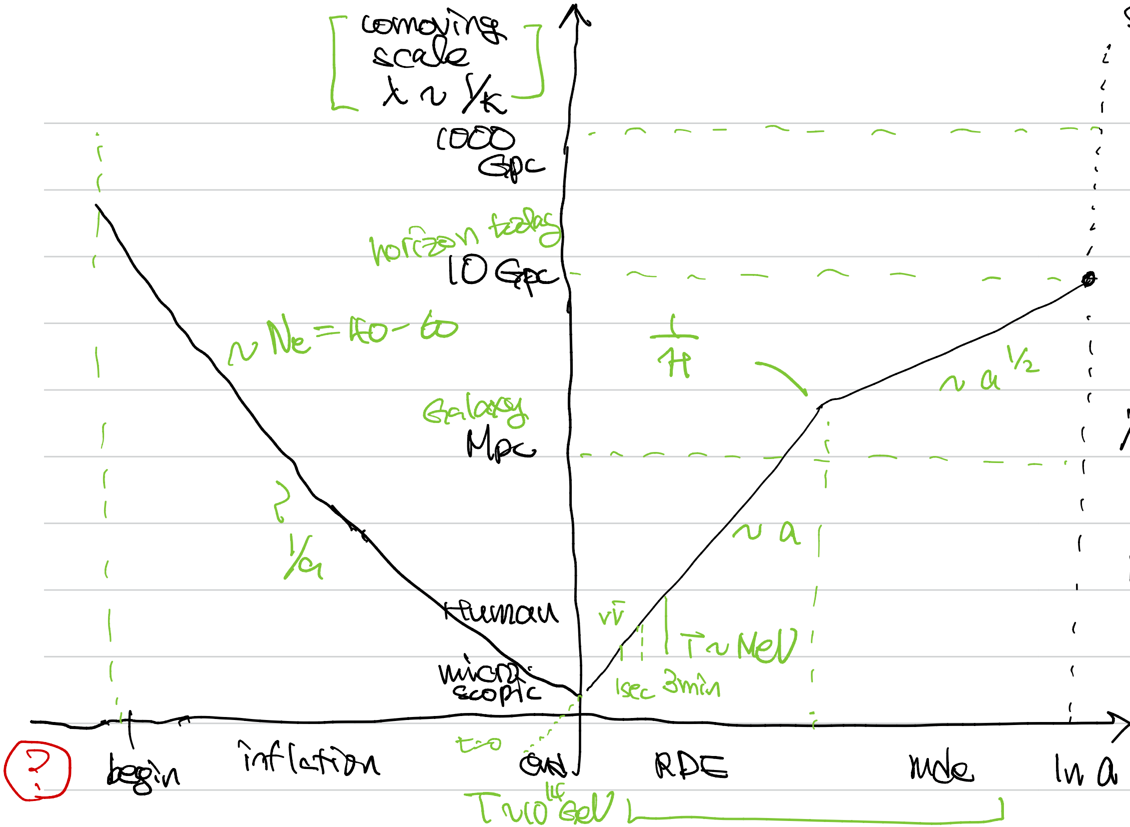


Standard Big Bang cosmo for general public

standard cosmology for scientists

$\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$

Conformal Hubble $\mathcal{H} = aH$

$k < \mathcal{H}$: super-horizon

$1/k > 1/\mathcal{H}$

today $a_0 \equiv 1$

$\lambda_{phy} = a \cdot \lambda_{com}$

$\lambda_{phy}(\text{today}) = \lambda_{com}(\text{today})$

physical 10 Gpc today $\rightarrow$ physical $\sim$ cm at a_{end}

$\rightarrow$ microscopic scale at a_{begin} ($N_e = 40$)

$H = \sqrt{\epsilon}\, 10^{14}\,\text{GeV} = \sqrt{\epsilon}\, 10^{-5} M_{pl}$ $\simeq \ln \frac{a_{end}}{a_{begin}}$

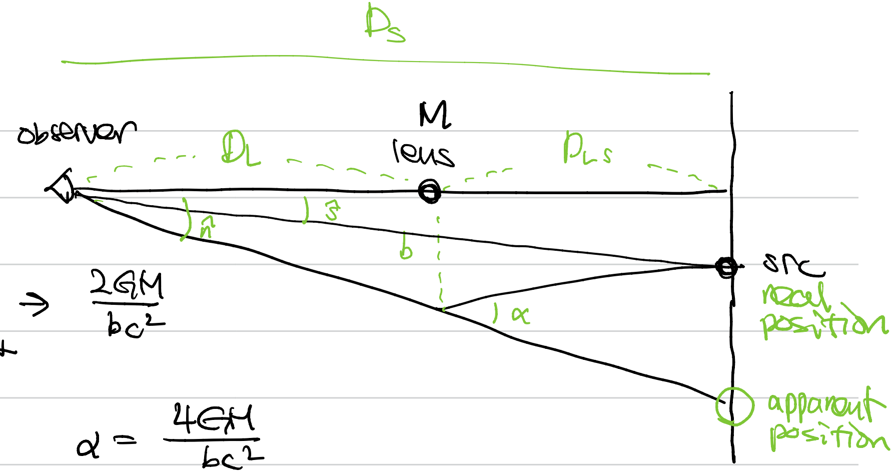
$\frac{1}{H} = \frac{10^5}{\sqrt{\epsilon}} t_{pl} \sim \frac{10^{-38}\,\text{sec}}{\sqrt{\epsilon}}$

$N_e = \int H dt = H \Delta t$

$\Delta t = \frac{N_e}{H} = N_e \cdot 10^{-37}\,\text{sec} \sim 10^{-35}\,\text{sec}$

Weak gravitational lensing

strong, weak, micro lensing



$$\Delta v_\perp = \frac{2GM}{b \cdot v_{rel}} \quad , \quad \alpha = \frac{2GM}{b \cdot v_{rel} \cdot \Delta v_\perp} \rightarrow \frac{2GM}{bc^2}$$

* relativistic correction : factor 2. $\quad \alpha = \frac{4GM}{bc^2}$

$$D_S \cdot \hat{s} = D_S \cdot \hat{n} - D_{LS} \cdot \alpha$$

$$\hat{s} = \hat{n} - \frac{D_{LS}}{D_S} \alpha = \hat{n} - \Delta \hat{n}$$ lens equation

$$b = D_L \cdot \hat{n}$$ $$\theta_E^2 = \frac{D_{LS}}{D_S D_L} \frac{4GM}{c^2}$$

point mass $\quad \hat{s} = \hat{n} - \theta_E^2 / \hat{n}$

Einstein radius $\quad \theta_E = 10^{-3} \text{ arcsec} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D_L}{\text{kpc}}\right)^{-1/2} \left(1 - \frac{D_L}{D_S}\right)^{1/2}$

$\hat{s} = \hat{n} - \Delta\hat{n}$ point mass $\rightarrow$ mass distribution or potential fluctuation $\rightarrow$ fluctuation over distance $\rightarrow$ src dist.

point mass $\Delta\hat{n} := \frac{D_{LS}}{D_S}\alpha$, $\alpha = \frac{4GM}{bc^2}$

$\hat{s} = \hat{n} - \nabla\Phi$, $\nabla\Phi = \Delta\hat{n}$, Φ : projected lensing potential

* mass distribution : $\Phi = \frac{1}{c^2}\frac{D_{LS}}{D_L D_S}\int dz \cdot 2\Psi$ cylindrical coordinate

$\nabla^2\Psi = 4\pi G\bar{\rho}\delta$, $\nabla^2\Phi = \frac{4\pi G}{c^2}\frac{D_L D_{LS}}{D_S}\int dz \cdot 2\delta\rho = 2\frac{\Sigma}{\Sigma_c}$, $\Phi = \int d^2n' \frac{\Sigma}{\pi\Sigma_c}\ln|n-n'|$

lensing kernel

* fluctuation over distance : $\Phi = \int_0^{r_s} dr_e \left(\frac{r_s - r_e}{r_s r_e}\right) 2\Psi = \int_0^{r_s} dr_e \frac{g(r_e, r_s)}{r_e^2} 2\Psi$ $g(r_e, r_s) = \frac{r_e(r_s - r_e)}{r_s}$

angular diameter distance

* src distribution : $\Phi = \int_0^{\infty} dr_e \frac{g(r_e, r_s)}{r_e^2} 2\Psi$. $g = r_e^2 \int_{r_e}^{\infty} dr_s \left(\frac{r_s - r_e}{r_s r_e}\right) n_g(r_s)$ $1 = \int_0^{\infty} dr_s\, n_g(r_s)$

$\hat{\nabla}\Phi$

$\hat{s} = \hat{n} - \Delta\hat{n}$: position , geodesic

$d\hat{s} = d\hat{n} - d\Delta\hat{n}$: shape , geodesic deviation

$ds_i = D_{ij}\, dn_j$. $D_{ij} = \frac{ds_i}{dn_j} = I_{ij} - \begin{pmatrix}\Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22}\end{pmatrix} = I - \begin{pmatrix}\kappa & 0 \\ 0 & \kappa\end{pmatrix} - \begin{pmatrix}\gamma_1 & \gamma_2 \\ \gamma_2 & -\gamma_1\end{pmatrix} - \begin{pmatrix}0 & \omega \\ -\omega & 0\end{pmatrix}$

$\kappa = 1 - \frac{1}{2}\mathrm{Tr}\, D = \frac{1}{2}(\Phi_{11} + \Phi_{22})$, $\omega = \frac{1}{2}(D_{21} - D_{12}) = -\frac{1}{2}(\Phi_{21} - \Phi_{12})$

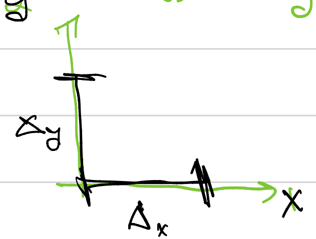
$\gamma_1 = \frac{1}{2}(D_{22} - D_{11}) = \frac{1}{2}(\Phi_{11} - \Phi_{22})$, $\gamma_2 = -\frac{1}{2}(D_{12} + D_{21}) = \Phi_{12} = \Phi_{21}$

$\Phi_{ij} = \hat{\nabla}_i \hat{\nabla}_j \Phi = \int dr_2\, g(r_1, r_2)\, \nabla_i \nabla_j \phi$

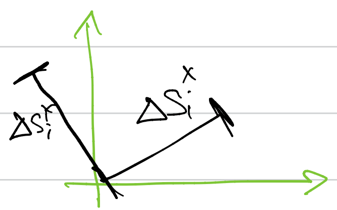
$\det D = (1-\kappa)^2 - \gamma^2 + \omega^2$

$\simeq 1 - 2\kappa$

obs. sky



2 src. coord



$M = \frac{dA_{obs}}{dA_{src}} = \det D^{-1}$

$= 1 + 2\kappa$

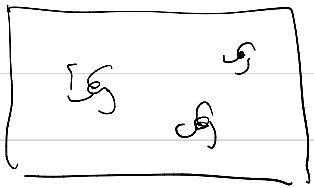
$dA_{obs} = \Delta_x \Delta_y$

$\Delta S_i^x = D_{i1}\, \Delta_x$

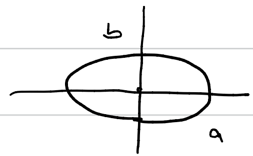
$\Delta S_i^y = D_{i2}\, \Delta_y$

$dA_{src} = D_{i1} D_{i2}\, \Delta_x \Delta_y$

$= \det D \cdot dA_{obs}$



obs

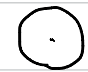


obs. galaxy
ellipse

lens

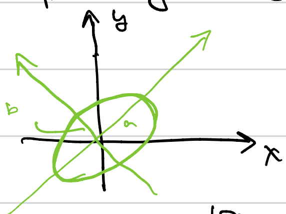
src

circle



galaxy

ellipticity $\epsilon^{obs} = \frac{a^2-b^2}{a^2+b^2} \doteq \delta + e^{int}$ $\quad q = \frac{b}{a} = 1-\delta$



$$\vec{\epsilon} \equiv (\epsilon_+, \epsilon_\times) = \left(\frac{M_{xx}-M_{yy}}{M_{xx}+M_{yy}}, \frac{2M_{xy}}{M_{xx}+M_{yy}} \right) = \epsilon(\cos 2\phi, \sin 2\phi)$$

$$ds_i = D_{ij}\, dn_j \quad \rightarrow \quad dn_i = (D^{-1})_{ij}\, ds_j$$

$$M^{obs}_{ij} \sim \int d^2n\, W(n)\, n_i n_j \sim \int d^2s\, |D^{-1}|\, W(s)\, (D^{-1})_{ik} s_k\, (D^{-1})_{jl} s_l$$

$$\sim (D^{-1})_{ik} (D^{-1})_{jl}\, |D^{-1}|\, M^{src}_{kl}$$

$$\epsilon_+^{obs} = \epsilon_+^{src} + 2\gamma_1 + O(2)$$

$$\epsilon_\times^{obs} = \epsilon_\times^{src} + 2\gamma_2 + O(2)$$

$$D^{-1} = \frac{1}{|D|} \begin{pmatrix} 1-\kappa+\gamma_1 & \gamma_2 \\ \gamma_2 & 1-\kappa-\gamma_1 \end{pmatrix}$$

$\epsilon_t^{obs} = \epsilon_t^{src} + 2\gamma_t$ no lensing $\epsilon_t = e \cos 2\Theta$ $e = 0 - 1$

$\epsilon_x^{obs} = \epsilon_x^{src} + 2\gamma_x$ $\gamma_1 = \gamma_2 = 0$ $\epsilon_x = e \sin 2\Theta$ $\Theta = 0 - 180^\circ$

1pt

$$\langle \epsilon_t \rangle \sim \frac{1}{N}\sum_i \epsilon_t(x_i) \sim \langle e \rangle \langle \cos 2\Theta \rangle = 0 = \langle \epsilon_x \rangle$$

$$\langle \epsilon_t \epsilon_x \rangle = \frac{1}{2}\langle e^2 \rangle \langle \sin 4\Theta \rangle = 0$$

$$\langle \epsilon_t^2 \rangle = \langle \epsilon_x^2 \rangle = \langle e^2 \rangle \langle \cos^2 2\Theta \rangle = \frac{1}{2}\langle e^2 \rangle$$

2pt $\langle \epsilon_t(x_1)\, \epsilon_t(x_2) \rangle$, $\langle \epsilon_x \epsilon_x \rangle$, $\langle \epsilon_t \epsilon_x \rangle$

$$\hookrightarrow = \cancel{\langle \epsilon^{src}_{t_1} \epsilon^{src}_{t_2} \rangle} + 4\langle \gamma_{t_1} \gamma_{t_2} \rangle + \cancel{\langle \epsilon^{src}_{t_1} 2\gamma_{t_2} \rangle} + \cancel{\langle \epsilon^{src}_{t_2} 2\gamma_{t_1} \rangle}$$

$\gamma \sim 10^{-4 - 5}$, $\epsilon^{src} \sim 0.3$

$$(\epsilon_+^{obs}, \epsilon_X^{obs}) \rightarrow (\gamma_+, \gamma_X) \sim \nabla_i \nabla_j \Phi \quad (\hat{n}) \rightarrow \Phi(\ell)\, \ell_i \ell_j \Rightarrow P_\Phi(\ell)$$

2D angular FT

$$\int d^2\hat{n}\, e^{i\ell\cdot n} \leftrightarrow \int d^2\ell\, e^{-i\ell\cdot n}$$