

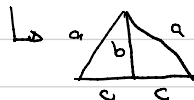
Motion of Conic Section

* Circle ($e=0$) . ellipse ($0 < e < 1$)

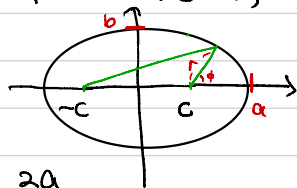
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

focus $c \equiv ae$

length of string = $2a$



$$\therefore b = a\sqrt{1-e^2}$$



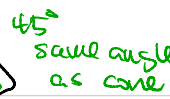
circle



ellipse



parabola



hyperbola

origin at focus.

$$\frac{R}{r} = 1 + e \cdot \cos(\phi - \phi_0)$$

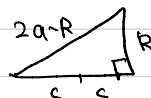
$$\phi = 90^\circ$$

$$r = R = a(1-e^2)$$

$$\phi = 0$$

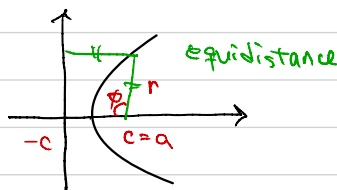
$$\frac{R}{r} = 1+e \quad r = a-c$$

$$= a(1-e) \quad \therefore R = a(1-e^2)$$



* parabola ($e=1$)

$$y^2 = 4ax$$



equidistance

$$\phi = 0^\circ \quad R = 2r = 2a$$

$$\phi = 90^\circ \quad R = r = 2a$$

$$\phi = 180^\circ \quad \frac{R}{r} = 0 \quad \therefore r \rightarrow \infty$$

* hyperbola ($e > 1$)

$$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$$

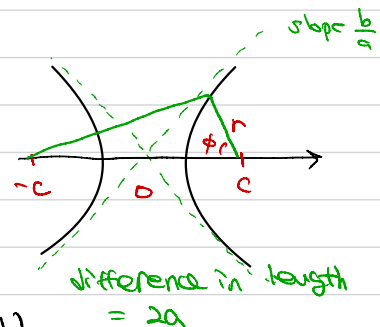
$$\phi = 0^\circ \quad \hookrightarrow x = a$$

$$\therefore r = a(e-1)$$

$$R = r(1+e) = a(e^2-1)$$

$$\phi = 90^\circ \quad r = R = \frac{b^2}{a} \quad x = ae$$

$$\therefore b = a\sqrt{e^2-1}$$



difference in length = $2a$