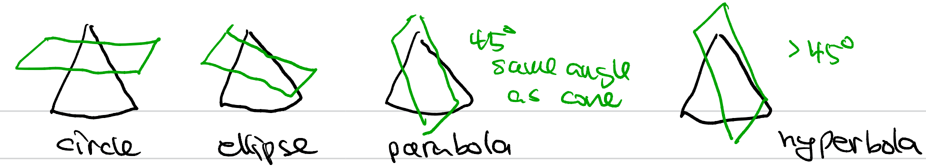
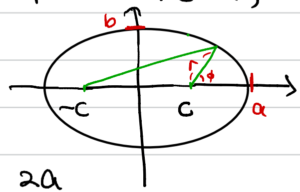
Motion of Conic Section



* circle $(e=0)$. ellipse $(0<e<1)$

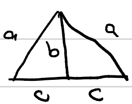
$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=1$$



focus $c \equiv ae$

length of string $= 2a$

↳

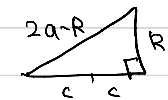


$\therefore\ b = a\sqrt{1-e^2}$

origin at Focus.

$$\frac{R}{r} = 1 + e\cdot\cos(\phi-\phi_0)$$

$\phi = 90°$. $r = R = a(1-e^2)$

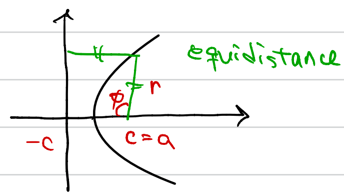


$\phi = 0$. $\frac{R}{r} = 1+e$. $r = a-c$

$= a(1-e)$ $\therefore\ R = a(1-e^2)$

* parabola $(e=1)$

$$y^2 = 4ax$$



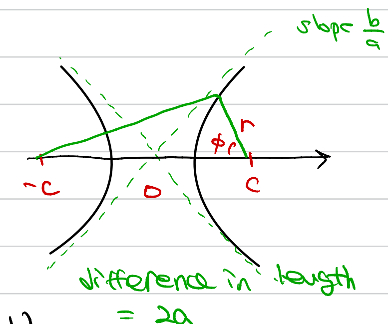
$\phi = 0°$, $R = 2r = 2a$

$\phi = 90°$, $R = r = 2a$

$\phi = 180°$. $\frac{R}{r} = 0$ $\therefore\ r \to \infty$

* hyperbola $(e>1)$

$$\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2=1$$



$\phi = 0°$ ↳ $x = a$

$\therefore\ r = a(e-1)$

$R = r(1+e) = a(e^2-1)$

$\phi = 90°$, $r = R = y$, $x = ae$

$\therefore\ b = a\sqrt{e^2-1}$