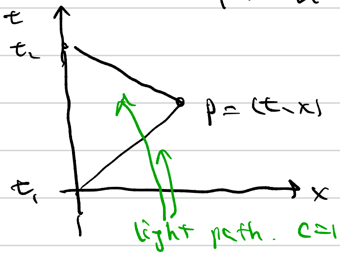
Lorentz transformation (derivation)

1) consider a spacetime diagram



then

$$t = \frac{1}{2}(t_1 + t_2)$$

$$x = \frac{c}{2}(t_2 - t_1)$$

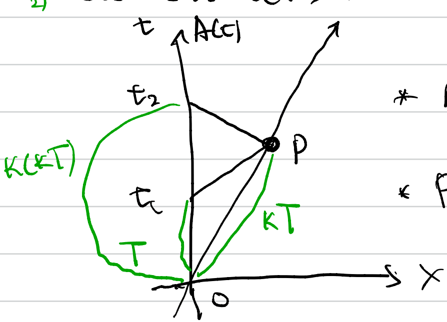
coord of A

or

$$t_1 = t - x$$

$$t_2 = t + x$$

2) now observers A & P. with clock synchronized at O



* A sends a signal at $t_1 \equiv T$

* P receives the signal at kT

k: some const. depends on v

in Newtonian

$$c(t - T) = v_p \cdot t$$

$$\therefore t = T \cdot \frac{c}{c - v_p} \rightarrow k$$

in proportion to T

* then for the same argument the duration (= t_2) should be $k \times (kT)$

$\therefore t_2 = k^2 T$

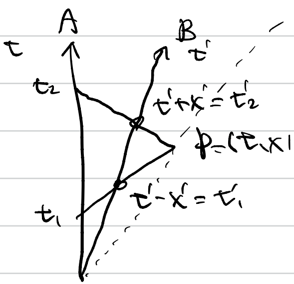
3) Therefore

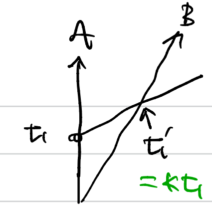
$$t = \frac{1}{2}(t_1 + t_2) = \frac{1}{2}T(1 + k^2)$$

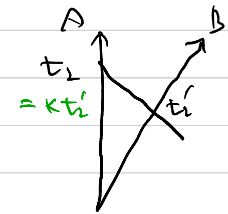
$$x = \frac{1}{2}(t_2 - t_1) = \frac{1}{2}T(k^2 - 1)$$

and $v_p = \frac{x}{t} = \frac{k^2 - 1}{k^2 + 1}$ & $k = \sqrt{\frac{1 + v}{1 - v}}$

4) now consider another observer B (moving relative to A)







$$\underline{t_1' = k\cdot t_1} \equiv k\cdot(t-x) = \overset{t_1'}{t'-x'}$$

$$\underline{t_2 = k\cdot t_2'} \equiv k\cdot(t'+x') = \underset{t_2}{t+x}$$

$$\therefore\ t' = \frac{1}{2}\left[k(t-x)+\frac{1}{k}(t+x)\right] = \gamma(t-vx)$$

$$x' = \frac{1}{2}\left[\frac{1}{k}(t+x)-k(t-x)\right] = \gamma(x-vt) \qquad \text{where} \quad \gamma = \sqrt{\frac{1}{1-v^2}}$$

5) now 3 frames w/ v_{AB}, v_{BC}.

since $K_{AC} = K_{AB}K_{BC}$. $\Rightarrow$ $\sqrt{\dfrac{1+v_{AC}}{1-v_{AC}}} = \sqrt{\dfrac{1+v_{AB}}{1-v_{AB}}}\sqrt{\dfrac{1+v_{BC}}{1-v_{BC}}}$

$$\therefore\ v_{AC} = \frac{v_{AB}+v_{BC}}{1+v_{AB}v_{BC}}$$