Issues in Standard Cosmology

$\chi = \int_e^0 \frac{dt}{a} = \int_{a_e}^1 \frac{d\ln a}{aH} = \int \frac{dz}{H}$ $\qquad -dt = a\,dr$. $\qquad d\ln a = -d\ln(1+z) = H\,dt$

* $\rho \sim a^{-n}$, $a \sim t^{2/n}$
  $\chi \sim [t^{-\frac{2}{n}+1}]_{t_i}^{t_f}$

* particle horizon $\chi_h = \chi(t_e = 0)$

for convergence at $a \to 0$. $\quad aH \propto \sqrt{\rho a^2 - K} \sim a^n \qquad n < 0$

* $\rho \sim \Lambda$. $a \sim e^{Ht}$
  $\chi \sim [-\frac{1}{H} e^{-Ht}]_{t_i}^{t_f}$

∴ ok for mde, rde, but no Λde

* event horizon $\chi_e = \chi(t_0 \to \infty)$

Yes for Λde, closed, but no for flat or open

Horizon problem

mde $\chi \simeq 6000 h^{-1}$ Mpc $(1+z)^{-1/2}$ $\rightarrow$ $\chi_h(z=1100) \sim 180\, h^{-1}$Mpc

comoving distance r to $z=1100$ $\rightarrow$ $r = \int_0^z \frac{dz}{H} \sim 5820\, h^{-1}$Mpc

$\rightarrow \theta \simeq 1.8$ degree $\simeq 0.031$ $\qquad$ no time for communication

Flatness problem

$\Omega_{tot} = \Omega_m + \Omega_r + \Omega_\Lambda + \cdots$ without Ω_K.

$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \rightarrow -\frac{3K}{8\pi G \rho a^2} = \frac{3H^2}{8\pi G \rho} - 1 = \Omega_{tot}^{-1} - 1$

$\rightarrow \frac{\Omega_{tot}^{-1}(t_{pl}) - 1}{\Omega_{tot}^{-1}(t_0) - 1} = \frac{(\rho a^2)_{eq}}{(\rho a^2)_{pl}} \frac{(\rho a^2)_0}{(\rho a^2)_{eq}} \sim \frac{T_0}{T_{eq}} \left(\frac{T_{eq}}{T_{pl}}\right)^2 \sim 10^{-60}$

today the deviation from unity $\Omega_{tot}^{-1} - 1 \simeq 10$

but smaller by 10^{-60} $\qquad$ ($K=0$ solved)

Monopole problem (real problem?)

$T_{GUT} \sim 10^{14-15}$ GeV . topological defects from phase transition

typically one defect per horizon at GUT $\rightarrow$ $R_{today} \sim T_{GUT}^4 \gg 10^{10}$

$\epsilon_0 \sim 10^{-7}$

Structure formation problem

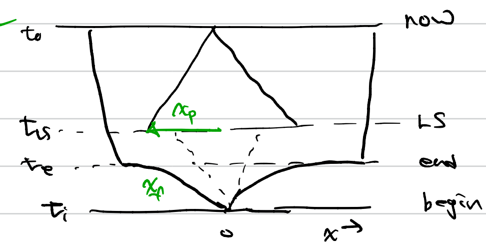
clusters of galaxies $M \sim 10^{15} M_\odot$, $R \sim 10$ Mpc comoving, $\epsilon_{grav} \sim 10^{-5}$ per unit proton mass

should be in causal contact to collapse gravitationally $\rightarrow$ $R < x_h$ $\rightarrow$ only $z < 10^6$ (or BBN)

but nothing in standard after BBN generates inhomogeneities of such high binding ϵ.

$\Rightarrow$ can be assumed as weird initial conditions

Inflation : accelerated expansion



$$x_p = \int_{t_{LS}}^{t_0} \frac{dt}{a} \sim 3t_0 \qquad \text{MDE}$$

$$x_f = \int_{t_i}^{t_f} \frac{dt}{a} = \frac{1}{a_e H_{inf}}\left[e^{H_{inf}\Delta t} - 1\right] \qquad a = a_e e^{H_{inf}(t-t_e)}$$

comoving horizon $\sim \frac{1}{aH}$ decrease $\leftrightarrow$ larger in the past

$$\frac{d}{dt}\left(\frac{1}{aH}\right) < 0$$

$$x_f \gg x_p \rightarrow e^{H\Delta t} > a_e H_{inf} \cdot 3t_0 \sim \frac{T_e}{T_0} = 10^{26}\left(\frac{T_e}{10^{14}\,\text{GeV}}\right) \Rightarrow H\Delta t \gtrsim 26 \ln 10 \simeq 60$$

60 e-folding

$$\Delta t \geq 10^{-33}\,\text{sec}$$

* $$\frac{\Omega_i^{-1} - 1}{\Omega_e^{-1} - 1} = \frac{(\rho a^2)_e}{(\rho a^2)_i} = \frac{a_e^2}{a_i^2} \geq (10^{26})^2 = 10^{52}$$

flatness problem mitigated by expansion

* scale for monopole $\sim \frac{1}{a^3} \sim e^{-3H\Delta t}$

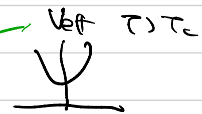
Higgs EW transition ~ 200 GeV
old inf. some GUT model
with Higgs-like potential

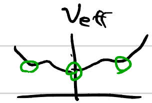
Requirement

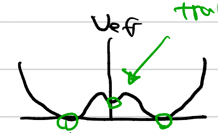
* has to end. (Λ : cannot).
* with large expansion. $T \to 0$ for ordinary radiation, if any.
  → reheating : needed & produce ordinary matter. inflaton gone
  hot enough for baryogenesis, but smooth enough for no monopole or defects

Models

* Old by Guth (original) $\boxed{\text{Higgs like}}$



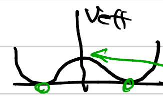




graceful exit problem
true vacuum : ordinary expansion
fall vacuum : inflation → large. all $V(\phi)$ is in kinetic energy of bubbles
they have to collide to release the energy & stop → too much inhomogeneities

Higgs ≠ inflaton

* New by Linde, Steinhardt (similar but



smooth 2nd-order phase transition. but slow-roll → minima ≫ M_{pl} unnatural.
also ϕ_{ini} should be $\ll M_{pl}$ otherwise stops too early → fine-tunning in initial condition

* chaotic by Linde : $V = \frac{1}{2} m_\phi^2 \phi^2$. chaotically some region satisfies $V \gg \rho$. slow-motion
  or any power-law potential → this region inflate → our univ.
  (other regions X) But this requires $\phi_{ini} \gg M_{pl}$. Quantum grav?
* other notable models : R^2 Starobinsky. power-law inf. axion-like model