Gauge

general measure . unit . or reference

best example is coordinates.

→ used for space time

gauge is more general than coordinate

→ typically used to measure symmetries

& coordinates for spacetime measure

e.g. once unit or reference is chosen. we can measure & assign # to any objects.

unit = cm → ⟝ = 2 (cm)

redundancy

such as U(1). SU(2)... in QFT

Poincare (Lorentz) . diffeomorphism

* reference $A(x)$ is chosen (gauge choice)
  → $\theta(x)$ for wind directions
* then another gauge choice $\tilde{A}(x)$
  gives $\tilde{\theta}(x) \neq \theta(x)$ for wind directions

gauge transformation

gauge in GR : diffeomorphism symmetry.

in Cosmology background reference FRW metric . $\bar{g}(t)$....

e.g. $g_{\mu\nu}(x) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(x)$

change of coord. $\tilde{x}^\mu = x^\mu + \xi^\mu$

$$\tilde{g}_{\mu\nu}(\tilde{x}) = \frac{\partial x^\rho}{\partial \tilde{x}^\mu}\frac{\partial x^\sigma}{\partial \tilde{x}^\nu} g_{\rho\sigma}(x) = \bar{g}_{\mu\nu}(\tilde{t}) + \delta\tilde{g}_{\mu\nu}(\tilde{x})$$

→ $\delta\tilde{g}(x) = \delta g_{\mu\nu}(x) + \cdots$

$$\delta_\xi T(x) \equiv \tilde{T}(x) - T(x) = e^{-\xi_\mu \partial^\mu} T \doteq -\xi_\mu T + \mathcal{O}(x)$$

→ $\delta_\xi \bar{T} = -\xi_\mu \bar{T}$