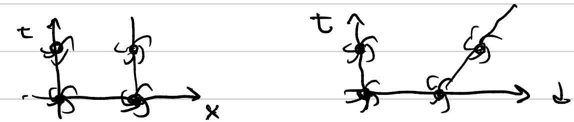
Redshift . Hubble law

To account for the expansion we use a comoving coordinate x

so that the physical separation $d = a(t)\, x$ ← only t

x: constant in homogeneous universe



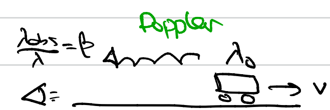
Hubble law

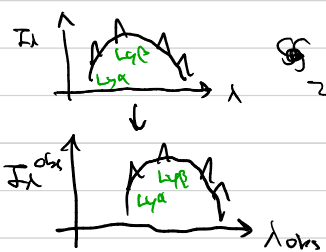
$\dot d = \dot a x = H d$ Hubble law $H = \frac{\dot a}{a}$: not constant

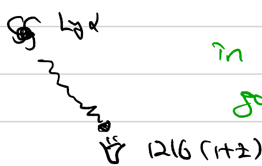
V_H = recession velocity

Redshift $1 + z = \frac{\lambda_{obs}}{\lambda_0}$ ∴ $z = \frac{\Delta\lambda}{\lambda_0} = V_z/c$ using Doppler effect ← Doppler velocity

↑ def of z







in fact due to the cosmic expansion. not due to the motion

galaxies not moving in homogeneous universe

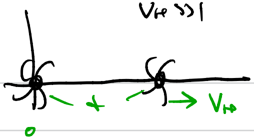
∴ not correct to use Doppler effect to interpret redshift.

$$z = \frac{\Delta\lambda}{\lambda_{rest}} \overset{SR}{=} \gamma(1+\beta_z) - 1 \Rightarrow \beta_z = \frac{(1+z)^2-1}{(1+z)^2+1} \simeq z$$

neither w/ special relativistic Doppler

$d \gg \frac{1}{H}$

$V_H \gg 1$



not on light cone

$V_H \ll 1$ for nearby observer

$V_H \gg 1$ for far away obs

observations

But at low $z \ll 1$, it reproduces the Hubble law

$$V_H = H d = H \frac{r_z}{1+z} \simeq \frac{cz}{1+z} \simeq cz = V_z \qquad r_z = \int_0^z \frac{dz}{H}$$

V_H for a given galaxy at x depends on observer

one cannot compare velocities at two different positions

Hubble law is valid all the time in homogeneous univ

V_H : recession velocity can be larger than c

not observable

solution of GR $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$

$g_{\mu\nu}$ from $R_{\mu\nu}$

Newtonian derivation of the Friedmann equation

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{K}{a^2} .$$

curvature : flat . closed . open

$K = 0, \pm 1$ by scaling the length unit R_0 (below)

↑ Hubble parameter

$\bar\rho(t)$ uniform density

consider a spherical region with M & R

cheating

$$\ddot R = -\frac{GM}{R^2}$$

only gravity. no pressure or other forces ∵ homogeneous.

integral constant

integrate → $\frac{1}{2}\dot R^2 = \frac{GM}{R} + E$

virial theorem → E : total energy

To accomodate the cosmic expansion

$R = a(t) R_0$, $M = \frac{4\pi}{3} R^3 \rho$, $E = -K R_0^2/2$

⇒ Friedmann equation

spacetime expansion. curved space. beyond Newtonian