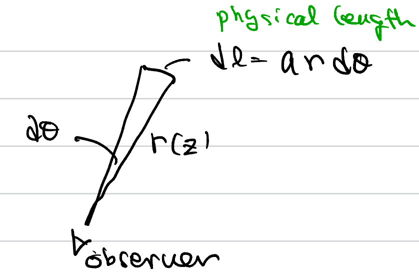
Cosmological Distance



* distance ladder : slides

* angular diameter distance $\quad dl = d_A \cdot d\theta \quad\quad d_A = a\, r_z$

$$ds^2 = 0 \Rightarrow a\,d\chi = dt = \frac{da}{\dot{a}} = -\frac{a}{H}\,dz \quad\quad \chi = \int_0^z \frac{dz}{H}$$

$$r_z = \begin{cases} \chi & K = 0 \\ \frac{1}{\sqrt{K}} \sin \sqrt{K}\,\chi_K & K > 0 \\ \frac{1}{\sqrt{-K}} \sinh \sqrt{-K}\,\chi_K & K < 0 \end{cases}$$

* standard ruler : physical length ← known

⇒ by measuring their angular size, we measure the angular diameter distance $\quad d_A = \frac{R}{d\theta}$

* look back time $\quad t_z = \int_0^z \frac{dz}{(1+z)H}$

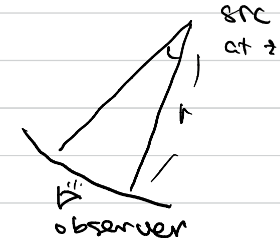
, $t(z=\infty)$ = age of the Univ.

* Luminosity distance $\quad f = L / 4\pi d_L^2 \qquad$ $d_L = a_0 r_z (1+z) = (1+z)^2 d_A$

in flat space, flux dilutes by $1/r^2$ (or area) given $d\Omega$
($\uparrow$ $E/T \cdot A \cdot \Omega$)

in Expanding univ. photon energy $\sim 1/a$. time interval $\sim 1/\nu \sim \lambda \sim a$
area $\sim a_0^2 r_z^2 d\Omega \sim d_A^2$

$$\therefore \frac{f}{L} = \frac{1}{4\pi (a_0 r_z)^2 (1+z)^2} \qquad \therefore d_L = a_0 r_z (1+z) = (1+z)^2 d_A$$



* standard candle : luminosity in the rest frame ← known
$\Rightarrow$ by measuring flux f, we measure luminosity distance