initial conditions at early univ

inflationary expansion : quantum fluctuations in vacuum → cosmological fluctuations

10 Gpc today → 1 cm at end of inflation → $< 10^{-15}$ m $N = 40 \sim 60$

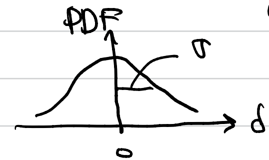
properties

* Gaussian & scale-invariant

∵ quantum fluctuations (more later)

* fluctuations w/ $\lambda(k)$ stretched to super-horizon scales & frozen. amplitude set by H

* nearly constant H → scale-invariant



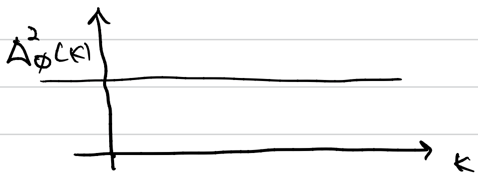
Gaussian

all at fixed k

only σ has all information

$\langle\delta\rangle = 0 = \langle\delta^3\rangle = \langle\delta^5\rangle = \cdots$

$\langle\delta^2\rangle \equiv \sigma^2$, $\langle\delta^4\rangle = 3\sigma^4$, ...



initial conditions

$\Rightarrow \sigma^2(k) \equiv P(k) \sim \langle|\delta_k|^2\rangle$ independent at each k

"power spectrum" how much power in each mode or energy scale

$$\Delta^2_\phi(k) \equiv \frac{k^3 P(k)}{2\pi^2} \equiv A\left(\frac{k}{k_0}\right)^{n_s-1}$$

dimensionless

amplitude A & spectral index $n_s \simeq 1$
$\sim 10^{-10}$

≃ gravitational potential

scale-invariant curvature power spectrum

$\Rightarrow k^2\phi \sim \delta_k$, $P_\phi \sim k^{n_s-4}$, $P_\delta \sim k^{n_s}$

but no matter after inflation

Probes of Inhomogeneity

galaxies, weak lensing, SN, CMB ...

$\delta(\vec{x})$: 3D field (great, difficult), 2D ($R_\perp$ or $\hat{n}$), 1D (along the line of sight)

easier to interpret → aliasing, mixing, limited info

Gaussian initial conditions $P(k)$

⇒ two-point correlation $\xi(r) = \langle \delta(\vec{x})\, \delta(\vec{x}+\vec{r}) \rangle_x$

∴ observations → $\delta(\vec{x})$

$$= \int D_{k_1} \int D_{k_2}\, e^{ik_1 x}\, e^{ik_2(x+r)}\, \underline{\langle \delta(k_1)\, \delta(k_2) \rangle} \quad = (2\pi)^3 \delta^D(k_1+k_2)\, P(k_1)$$

→ $\xi(r)$, $P(k)$ compare

$$= \int D_k\, e^{ikr}\, P(k) = \int d\ln k \left(\frac{k^3 P}{2\pi^2} \right) j_0(kr)$$

dimensionless

Δ_k^2 : cont to σ^2 per $\ln k$

galaxy # density $n_g = \bar{n}_g (1 + \delta_g)$

$$\langle n_g(\vec{x})\, n_g(\vec{x}+\vec{r}) \rangle = \bar{n}_g^2 \left[1 + 2\langle \delta_g \rangle + \langle \delta_g(x)\, \delta_g(x+r) \rangle \right]$$
$$= \bar{n}_g^2 \left[1 + \xi_g(r) \right]$$

excess probability

same for 3pt & Bispectrum

4pt & Trispectrum