Physical Cosmology

Problem set 2

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Exercise 1 (5 points)

Consider the FLRW metric given by the line element

$$ds^2 = -\mathrm{d}t^2 + a(t)^2 \left(d\chi^2 + S_k^2(\chi) d\Omega^2 \right) \,, \tag{1}$$

and the expression for the luminosity distance

$$d_L = (1+z) a_0 S_k(\chi) \,, \tag{2}$$

where a_0 is the scale factor today.

We want to study three supernovae explosions that we observe at redshift $z = \{0.02\,, 0.2\,, 2\}$.

- Compute the comoving distance χ out to redshift z by integrating a light-like separation. Show that the final expression takes the form

$$\chi(z) = \int_0^z \frac{\mathrm{d}z'}{H(z')} \,. \tag{3}$$

  Write $H(z)$ in terms of the Hubble constant H_0 and the cosmological density parameters at redshift zero: $\Omega_{m0}, \Omega_{\gamma 0}, \Omega_{K0}, \Omega_\Lambda$

- Integrate the Hubble rate $H(z)$ to derive an (integral) expression for the elapsed cosmic time between the explosion and detection of the supernova

- Compute numerically the luminosity distances if the explosions take place in a flat Universe ($K = 0$) with $\Omega_{m0} = 0.3$, $\Omega_\Lambda = 0.7$ (neglect the other density parameters). Take $H_0 = 72\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Compute also the elapsed cosmic time between explosion and detection. Express the results in gigaparsec (Gpc) and gigayears (Gyr)

- Repeat the computations of the previous point but for an open Universe ($K < 0$) with $\Omega_{m0} = 0.3$ and $\Omega_\Lambda = 0$

Exercise 2 (5 points)

The surface brightness (also known as intensity) of an astronomical object is defined as

$$I = \frac{F}{\Omega} \,, \tag{4}$$

where Ω is the observed solid angle subtended by the source, and $F = \frac{L}{A}$ is the flux emitted by the source of bolometric (integrated over all frequencies) luminosity L over an area A.

- Express the solid angle Ω in terms of the angular diameter distance, and the flux F in terms of the luminosity distance. How does the surface brightness I scale with redshift?

- Assume that an astronomical object is both a "standard candle" and a "standard ruler" – hence, you can measure both the luminosity distance and the angular diameter distance. Show that this kind of observation rules out the "tired light model".

  Hint: After astronomers such as Edwin Hubble have shown that distant objects appear more redshifted, Swiss astronomer Fritz Zwicky proposed the "tired light model" in 1929. In that model, photons would collide with other particles and (over cosmic time scales) lose energy. In such a model, how would the luminosity distance and the angular diameter distance scale with redshift?