Physical Cosmology

Problem set 4

Spring Semester 2025
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Issued: 21.3.2025
Due: 28.3.2025

Exercise 1 (4 points)

In the very early Universe, free protons and free neutrons were in thermal equilibrium with the hot plasma. The chemical equilibrium was maintained through reactions with electronic neutrinos and antineutrinos:

$$n + \nu_e \leftrightharpoons p + e^- \,, \qquad p + \bar{\nu}_e \leftrightharpoons n + e^+ \,. \tag{1}$$

Determine the freeze-out temperature of these reactions and compute the ratio between the number density of free neutrons and protons at freeze-out. Does this ratio evolve after freeze-out? Motivate your answer.

Exercise 2 (6 points)

a) **Hot relics:** The exact mass $\sum m_i$ of the neutrinos (summed over the three different flavors) is not known. Lab experiments can provide both lower and upper bounds on the mass. However, the best upper bounds are obtained from cosmological constraints.

- Neutrinos were relativistic when they decoupled. Assuming that they are non-relativistic today, compute their relic contribution to the energy density $\rho_{\nu_i,0}(m_i)$ as a function of the neutrino mass of a given flavor m_i
- Assuming that the neutrinos do not over-close the universe ($\sum_i \rho_{\nu_i,0} \leq \rho_{\mathrm{crit},0}$), show that an upper bound on the neutrino masses is given by

$$\sum_{i=1,2,3} m_i \leq 93\, h^2 \,\mathrm{eV} \,, \tag{2}$$

where $H_0 = 100h\,\mathrm{s}^{-1}\,\mathrm{km/Mpc}$

b) **Cold relics:** The lightest neutralino species predicted by supersymmetry should have a mass of at least 30 GeV, based on various experimental constraints provided by LHC

- What is the expected contribution to the density parameter of such relic, and would it be consistent with cosmological constraints?
- Explain how the decoupling time changes as the mass of the particle increases

Hint: Read pages 135–137 in "Galaxy Evolution and Formation" to answer this question.