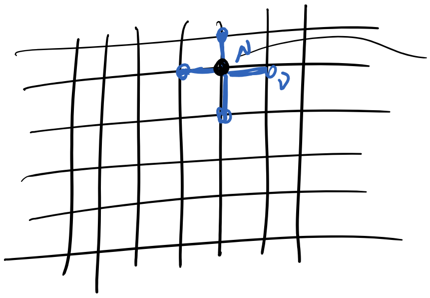
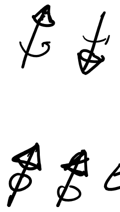
2-D Ising Model

Monday, 4 April 2022 12:34



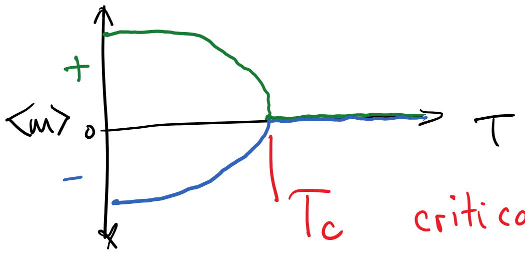
$S_\nu = \pm 1$



N spins $\nu = 1 .. N$ on a uniform grid of constant spacing.

lower energy

What happens as we change the temperature as $T \to 0\,K$



critical temperature

2nd order Phase transition

$$H = -J \sum_{\substack{\text{all } \nu \\ \text{and } \mu \in \text{Neighbor}(\nu)}} S_\nu S_\mu$$

the product $S_\nu S_\mu$ is either $+1$, -1

aligned anti-aligned

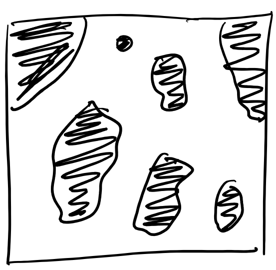
If $J > 0$ Ferromagnetic behaviour

$J < 0$ Antiferromagnetic behaviour

For this 2-D Ising model an exact solution was found by Onsager (1944).

$$\frac{J}{k_B T_c} = -\frac{1}{2} \log\left(\sqrt{2} - 1\right) \cong 0.44069\ldots$$

What does it look like $0 < T < T_c$?



Large connected regions of the same spin

magnetic field Σ of the spins.

$$\langle m \rangle = \int_{\substack{\text{All } C_i \\ \text{possible} \\ \text{states}}} P(C_i)\, M(C_i)\, dC_i$$

12×12 lattice

$$\underbrace{2^{144}}_{\text{states}} \cdot 10^{-16}\,s = 10^{17}\,s$$

longer than the age of the Universe!

Pick a path through configuration space.

Where we are interested only in the equilibrium behaviour (e.g., $\langle m \rangle$)

Special path through this configuration space when averaged over gives you the $\langle m \rangle$ in equilibrium.

$$C_1 \xrightarrow[\text{spin}]{\text{flip a}} C_2 \xrightarrow[\text{spin}]{\text{flip a}} C_3 \cdots \rightarrow C_s$$

Must obey <u>detailed balance</u>

transition probability

$$\frac{W(+S_\nu \rightarrow -S_\nu)}{W(-S_\nu \rightarrow +S_\nu)} = e^{-2\beta J s_\nu \sum_{N \in (\nu)} S_N}$$

(in equilibrium)

$$\beta = \frac{1}{k_B T}$$

If the move $+S_\nu \rightarrow -S_\nu$ goes to a lower energy $W = 1$, always accept.

If the move $+S_\nu \rightarrow -S_\nu$ goes to higher energy (ΔE is +ve), do $+S_\nu \rightarrow -S_\nu$ with a probability $\underbrace{e^{-\beta \Delta E}}$.

Pick a random number $[0,1)$

Metropolis Algorithm

$$w(+s_\nu \to -s_\nu) = \min(1, e^{-\beta \Delta E})$$
$$w(-s_\nu \to +s_\nu) = \min(1, e^{\beta \Delta E})$$

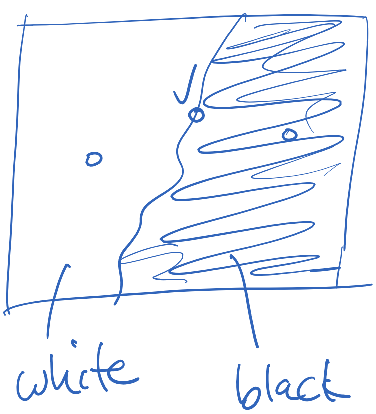
Algorithm

1. Pick a (random) spin s_ν
2. Calculate ΔE for flipping
3. If $\Delta E < 0$ —— $\boxed{s_\nu = -s_\nu}$
4. else
   - 4a. Pick a random $0 \leq r < 1$
   - 4b. If $r < \exp(-\beta \Delta E)$
5. otherwise no change

Start with a high T and visualize the result

$T > T_c$ $\quad \langle m \rangle = 0$ "disorder"

$T < T_c$ $\quad$ correlated spins "order" and $m \neq 0$.



$\boxed{\text{40 × Number of spins should be tried.}}$