Population Growth, Chaos & Fractals

Monday, 2 October 2023 12:34

Constant growth rate for a population

$$\frac{P_{n+1} - P_n}{P_n} = r \qquad P_n(P_0) = ?$$

or

$$\boxed{P_{n+1} = (1+r)P_n} \qquad P_{n+1} = (1+r)(1+r)P_{n-1}$$

$$= (1+r) \cdots (1+r)P_0$$

$$\boxed{P_{n+1} = (1+r)^n P_0}$$ "population explosion"

Typically: Resources are finite

Normalize the population with respect to some "sustainable" size for the population, N

$$\boxed{p = P/N}$$

p	r
1	0
small	large, positive
~ 1	small
>1	negative

$$r \propto (1-p) \Longleftarrow$$

$$r = k(1-p)$$

Verhulst Model

$$\frac{p_{n+1} - p_n}{p_n} = k(1-p_n)$$

$$p_{n+1} = p_n + k\,\boxed{p_n}(1-\boxed{p_n}) \qquad \propto p_n^2$$

Non-linear!

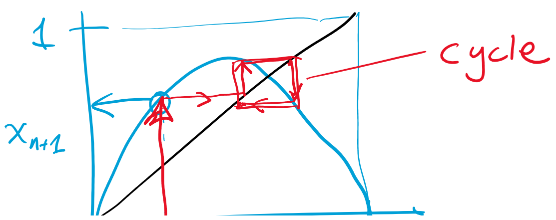
E.g. $k=3$ $\quad p_0 = 0.01$

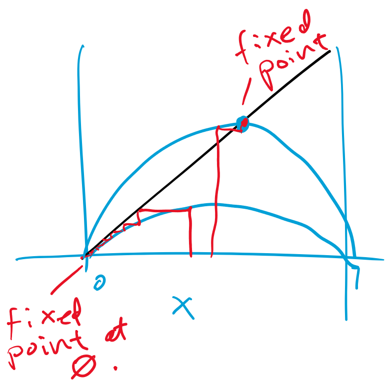
$p_0 = 0.0099999$

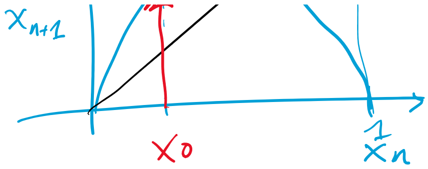
$$x_{n+1} = \boxed{a x_n(1-x_n)} \qquad \text{Logistic Equation}$$

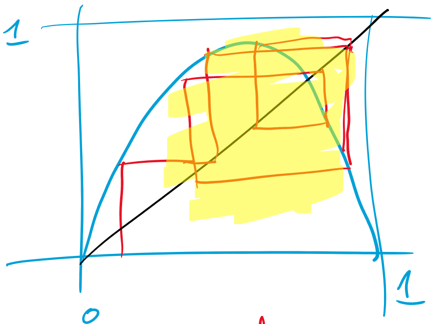
$$x \in [0,1]$$

$$a \in [0,4]$$

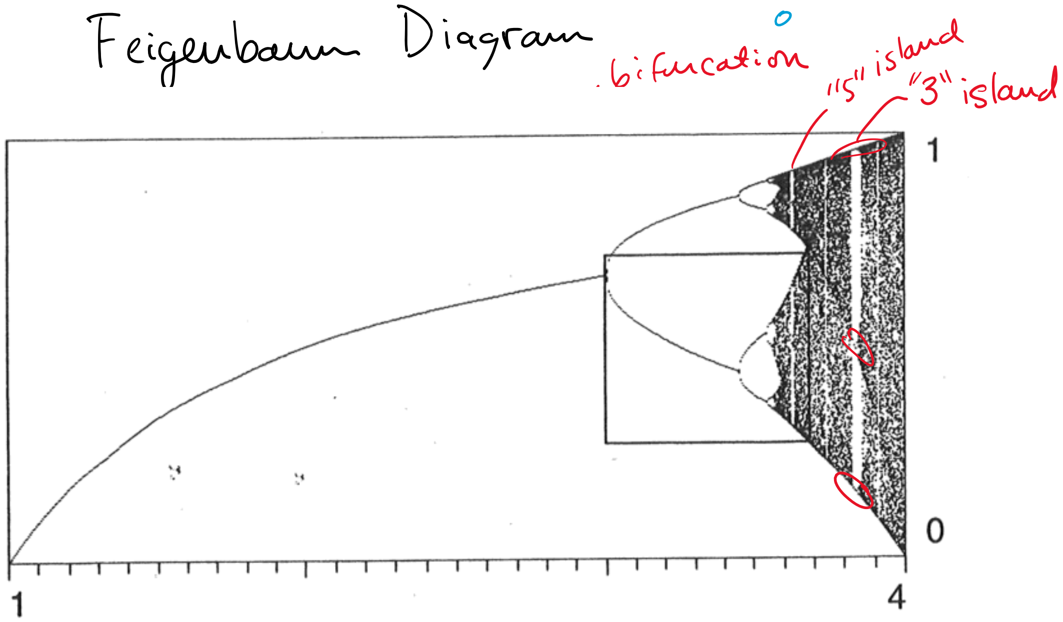


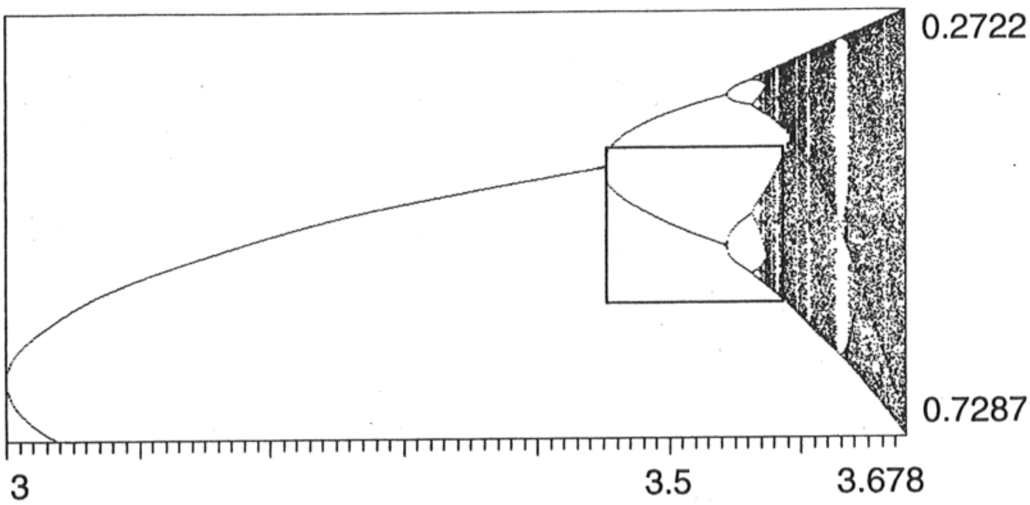


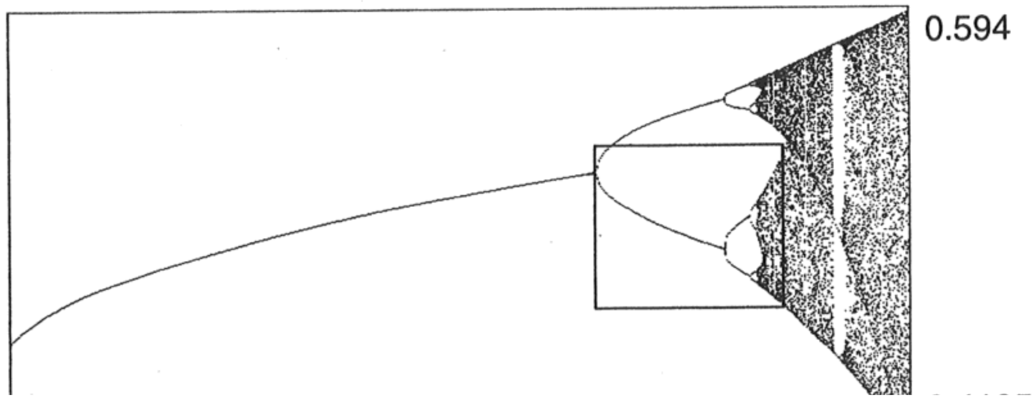


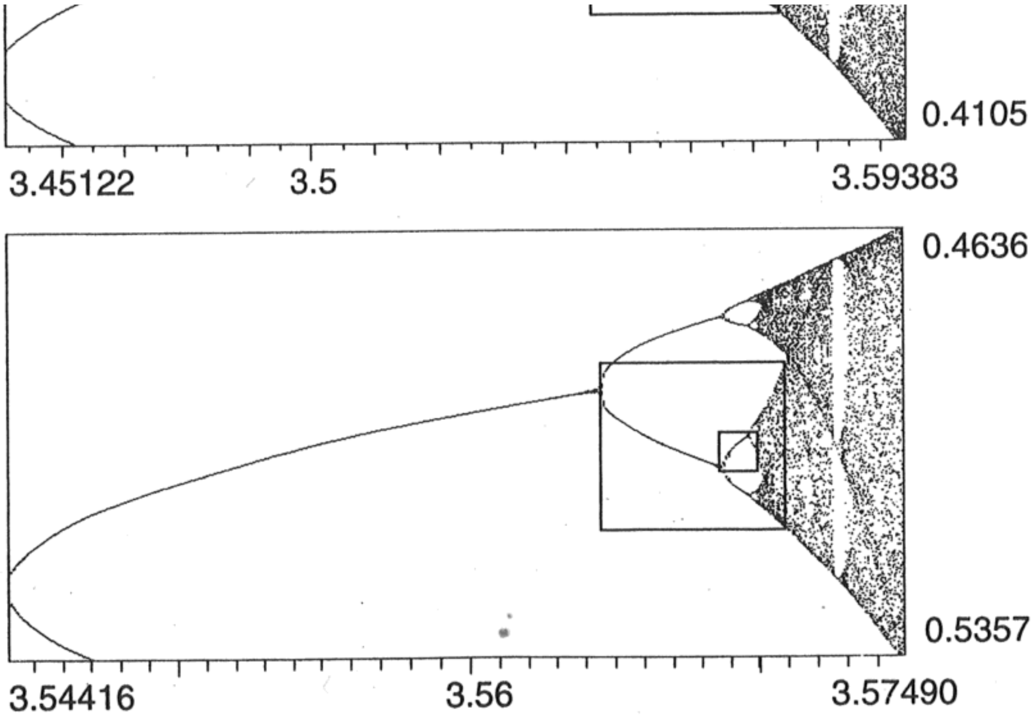


Feigenbaum Diagram

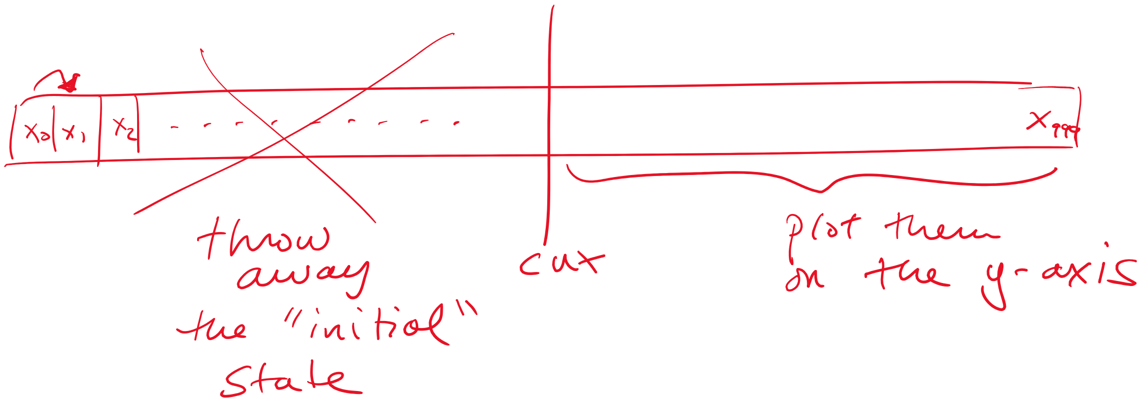




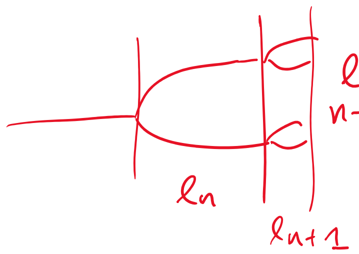




Non-linear ⇝ Fractal behaviour



Universality:



$$\lim_{n\to\infty} \frac{l_n}{l_{n+1}} = 4.6692\ldots$$