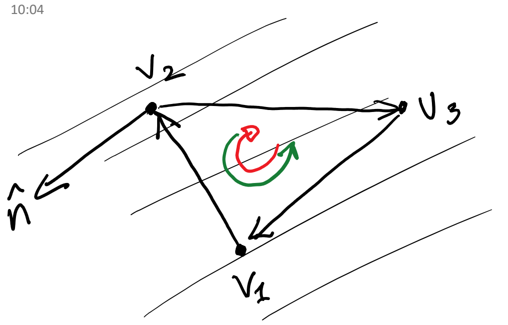
Ray Tracing - Triangulations

Monday, 6 March 2023 10:04



consistent handedness

$$\hat{n} \cdot \underline{v}_i + D = 0 \quad \text{plane-equ.}$$

$$D = -\hat{n} \cdot \underline{v}_i$$

$$\hat{n} \cdot \underline{p} - (\hat{n} \cdot \underline{v}_i) = 0$$

then p is in the plane

$$\underline{n} = (\underline{v}_2 - \underline{v}_1) \times (\underline{v}_3 - \underline{v}_1)$$

Some ray $\underline{r} = \underline{d}\,t + \underline{o}$, put this into the plane equation

$$(\hat{n} \cdot \underline{d})t + (\hat{n} \cdot \underline{o}) + D = 0$$

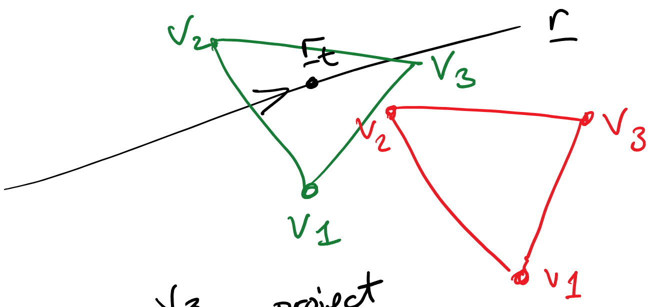
Solve for t:

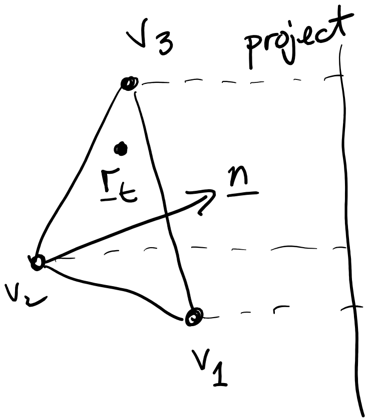
$$t = \frac{-(\hat{n} \cdot \underline{o}) - D}{(\hat{n} \cdot \underline{d})} \longleftarrow \text{check for } 0!$$

Is t in a reasonable range?

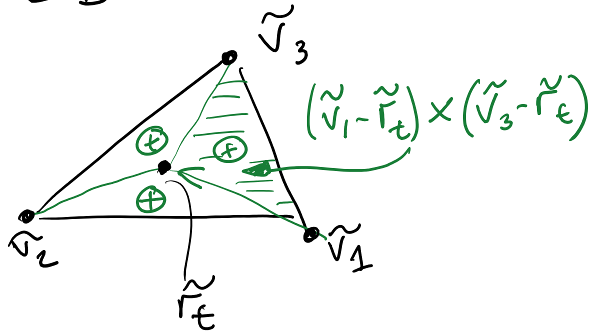
$t \in [0, 1]$ maybe

$\underline{r}_t$ is a possible point in the triangle

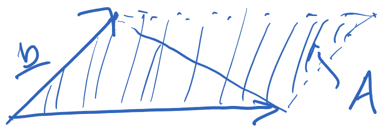




in 2-D



x-direction has the largest component of $\hat{n}$



$A = |\underline{a} \times \underline{b}|$

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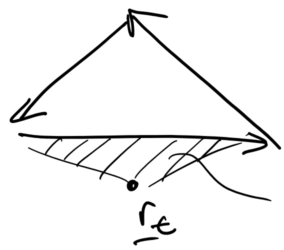
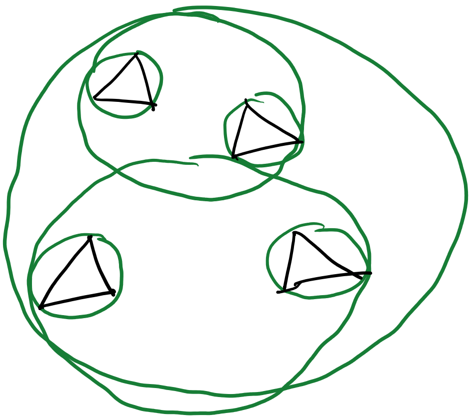
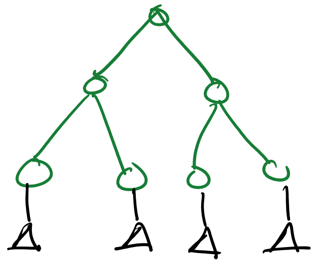
If all 3 sub-triangles are $\oplus$ cross product then $\underline{r}_t$ is inside the triangle
or all are $\ominus$ $\Rightarrow$ $\underline{r}_t$ inside







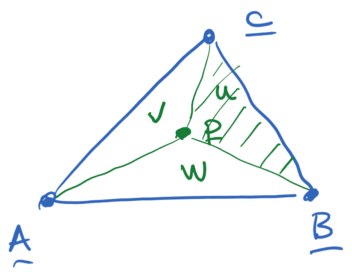
Bounding Volume Hierarchy

And we could have used boxes.

AABB - axis-aligned bounding box

Recursive Treewalk algorithm.

Barycentric Coordinates for the triangle



$$\underline{P} = u\,\underline{A} + v\,\underline{B} + w\,\underline{C}$$

$u + v + w = 1$ (normalized area)

$$w = 1 - u - v$$

$$u + v \leq 1$$

$\underline{P}$ is in the triangle if

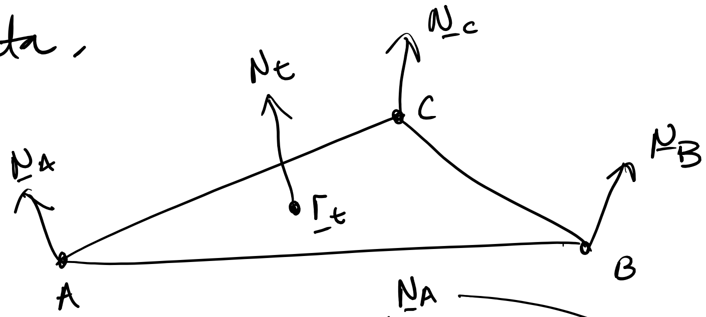
$$0 \leq u, v, w \leq 1$$

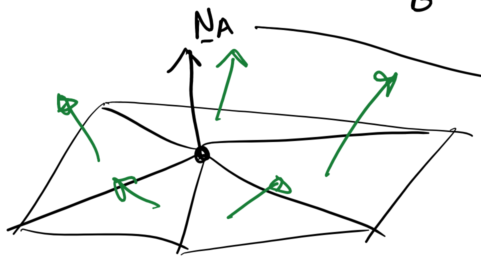
"Areal Coordinates"

$$\text{Triangle Area} = \frac{|\underline{B}-\underline{A}| \cdot |\underline{C}-\underline{A}| \sin\theta}{2}$$

$$= \frac{|(\underline{B}-\underline{A}) \times (\underline{C}-\underline{A})|}{2}$$

This allows the interpolation of any Vertex Data.





Average normal of all neighbor triangles.

$$\underline{N} = \frac{\sum_i \text{Area}_i \cdot \underline{N}_t}{\sum_i \text{Area}_i}$$