We used $\underline{u}$ denote fluid velocity, but I want to switch now to using $\underline{V}$ which will be an added field of every particle.

$$\nabla \cdot \underline{V} \cong \sum_b \frac{m_b}{\rho_b} \underline{V}_b \cdot \nabla W(|\underline{r} - \underline{r}_b|, h)$$

size of the Kernel.

Monahan cubic-spline kernel

We could use this, but ...

$$\nabla \cdot \underline{V} \equiv \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot \left(V_x \hat{i} + V_y \hat{j} + V_z \hat{k}\right)$$

$$= \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

$$\nabla \cdot \underline{V} = \frac{1}{\rho}\left[\nabla \cdot (\rho \underline{V}) - \underline{V} \cdot \nabla \rho\right]$$

$$\nabla \cdot \underline{V}_a \cong \frac{1}{\rho_a}\left[\sum_b m_b \frac{\cancel{\rho_b} \underline{V}_b}{\cancel{\rho_b}} \cdot \nabla_a W_{ab} - \underline{V}_a \cdot \sum_b m \nabla_a W_{ab}\right]$$

$$\rho_a (\nabla \cdot \underline{V}_a) \cong \sum_b m_b (\underline{V}_b - \underline{V}_a) \cdot \nabla_a W_{ab}$$

$W(|\underline{r}_a - \underline{r}_b|, h)$

$$\frac{de}{dt} = -\frac{P}{\rho} \nabla \cdot \underline{V}$$

switched $a \leftrightarrow b$ -1!

energy

$$\boxed{\frac{de_a}{dt} = \left(\frac{P_a}{\rho_a^2}\right) \sum_b m_b (\underline{V}_a - \underline{V}_b) \cdot \nabla_a W_{ab}}$$

Benz Formulation

continuity

$$\rho_a = \sum_b m_b W_{ab}$$

(Euler equation) $$\frac{d\underline{v}_a}{dt} = -\frac{\rho_a \nabla P_a}{\rho_a^2}$$

$$\frac{d\underline{v}_a}{dt} = -\frac{1}{\rho_a^2}\sum_b m_b \left(P_b - P_a\right) \nabla_a W_{ab}$$

Force $\rightarrow 0$ for const. Pressure, but linear momentum is not conserved and angular momentum is also not conserved.

Instead: $$\frac{\nabla P}{\rho} = \nabla\left(\frac{P}{\rho}\right) + \frac{P}{\rho^2}\nabla\rho$$

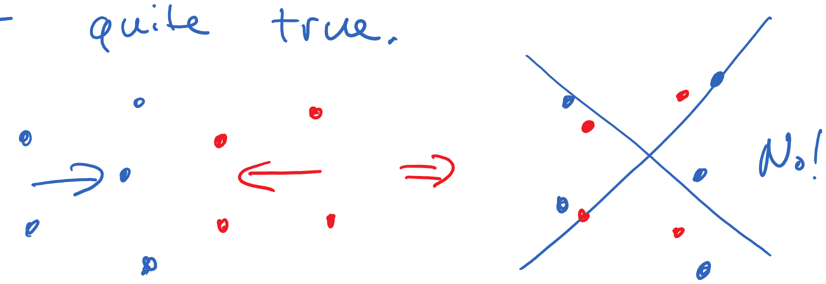
$$\boxed{\Rightarrow \frac{d\underline{v}_a}{dt} = -\sum_b m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2}\right) \nabla_a W_{ab}}$$

Looks symmetric $a \Leftrightarrow b$

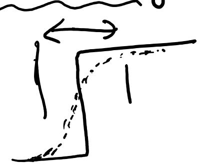
Newton's 3rd Law: $F_{ab} = -F_{ba}$

momentum conserved

Caveat: However if particles a and b have differing kernel sizes h_a, h_b then this is not quite true.



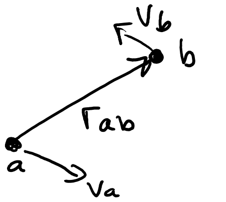
Add Artificial Viscosity



Shock

$$\frac{d\underline{v}_a}{dt} = -\sum_b m_b \left(\underbrace{\frac{P_a}{\rho_a^2}}_{c^2/\gamma\rho} + \frac{P_b}{\rho_b^2} + \Pi_{ab}\right) \nabla_a W_{ab}$$

$dt \quad b$



$$\Pi_{ab} = \begin{cases} -\alpha \, \bar{c}_{ab} \, \mu_{ab} + \beta \, \mu_{ab}^2 , & \underline{v}_{ab} \cdot \underline{r}_{ab} < 0 \\ 0 , & \underline{v}_{ab} \cdot \underline{r}_{ab} > 0 \end{cases}$$

$$\mu_{ab} = \frac{\bar{h}_{ab} \, \underline{v}_{ab} \cdot \underline{r}_{ab}}{r_{ab}^2 + \eta^2} \qquad \bar{h}_{ab} - \tfrac{1}{2}(h_a + h_b)$$

$\beta = 2\alpha$
$\alpha = 1, \ \beta = 2$
$\alpha = 0.5, \ \beta = 1$

small number to keep it from exploding.

$$e_a = \frac{P_a}{\rho_a(\gamma - 1)} \qquad c_a = \sqrt{\gamma(\gamma-1)\, e_a}$$

$$\frac{P}{\rho^2} = \frac{c^2}{\gamma \rho}$$

Let's "write" the code:

Variables for particles: $\underline{r}, \underline{v}, e, c, \rho, h^{NN}$

we need to calculate $\underline{a} = \frac{d\underline{v}}{dt} \quad \dot{e} = \frac{de}{dt}$

☆ need 2 extra temporary variables → ODE "Leapfrog"

$\underline{v}_{pred}$, e_{pred}

DRIFT1(), DRIFT2(), KICK(), CALCFORCE()

$\underline{v}_{pred}$, e_{pred}

SPH:

```
DRIFT1(Δt=0)        vpred  epred
CALCFORCE()
for (step = 0; step < NSTEP; ++step) {
    DRIFT1(Δt/2)
    CALCFORCE()
```

KICK(Δt)

DRIFT2($\Delta t/2$)

}

CALCFORCE:

TREEBUILD

NN-Density ← all particles calculate ρ_a

CALCSOUND ← all particles calculate c

$c_a = \sqrt{\gamma(\gamma-1)\, e_{pred}}$

NN-SPHFORCE ← All calculate $\underline{a}$, $\dot{e}$

DRIFT1(Δt):

$\underline{r} \mathrel{+}= \underline{v}\,\Delta t$

$\underline{v}_{pred} = \underline{v} + \underline{a}\,\Delta t$

$e_{pred} = e + \dot{e}\,\Delta t$

DRIFT2(Δt):

$\underline{r} \mathrel{+}= \underline{v}\,\Delta t$

KICK(Δt):

$\underline{v} \mathrel{+}= \underline{a}\,\Delta t$

$e \mathrel{+}= \dot{e}\,\Delta t$

$$\overline{W}_{ab} = \frac{1}{2}\left(W(r_{ab}, h_a) + W(r_{ab}, h_b)\right)$$

Symmetrized the Kernel

How is this done?