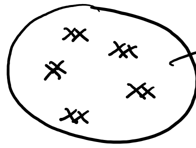
Kernels and a "Wind Tunnel"

29 March 2021 12:04

Kernels: Monahan "clumping instability" for larger number of neighbors $n > 80$

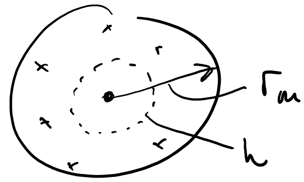


Kernel Sampling is ruined.

Wendtland Kernels: Quintic

$$f(q) = \alpha_\sigma \begin{cases} \left(1 - \frac{q}{2}\right)^4 (1 + 2q), & 0 \leq q \leq 2 \\ 0, & 2 < q \end{cases}$$

$$q = \frac{|r_{ab}|}{h} \qquad h = \frac{r_m}{2}$$



Normalizing factors $\alpha_1 = \frac{3}{4h}$, $\alpha_2 = \frac{7}{4\pi h^2}$, $\alpha_3 = \frac{21}{16\pi h^3}$

For the gradient

$$f'(q) = \frac{\alpha_\sigma}{h} \begin{cases} -5q\left(1 - \frac{q}{2}\right)^3, & 0 \leq q \leq 2 \\ 0, & 2 < q \end{cases}$$

Should solve the clumping instability problem for large number of neighbors.

Symmetrizing of the Kernel: Conserve Momentum

$$\underline{\left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2}\right)} \checkmark$$

$$W_{ab} = W(|r_{ab}|, \underline{h_a})$$

$$h_{ab} = \frac{1}{2}(h_a + h_b) \checkmark \quad \text{difficult to implement}$$

Easier way: $W_{ab} = \frac{1}{2}\left(W(r_{ab}, h_a) + W(r_{ab}, h_b)\right).$

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Symmetrize the Kernel ✓

(1) $$\frac{d\underline{v}_a}{dt} \mathrel{+}= \boxed{-\frac{1}{2}\sum_b m_b \underbrace{F_{ab}}_{\left(\frac{P_a}{\rho_a^2}+\frac{P_b}{\rho_b^2}+\Pi_{ab}\right)} \nabla_a W(r_{ab}, h_a)} \checkmark$$

(2) $$\frac{d\underline{v}_a}{dt} \mathrel{+}= -\frac{1}{2}\sum_b m_b \boxed{F_{ab}\nabla_a W(r_{ab}, h_b)}$$

gather part

(2) rewrite the $a \leftrightarrow b$

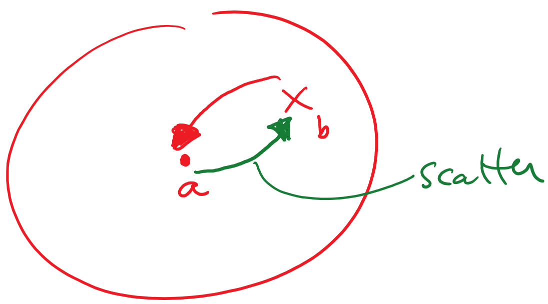
$$\frac{d\underline{v}_b}{dt} \mathrel{+}= -\frac{1}{2}\sum_a m_a F_{ab}\nabla_b W(r_{ab}, h_a)$$

but $\nabla_b = -\nabla_a$

same!

$$\frac{d\underline{v}_b}{dt} \mathrel{+}= \frac{1}{2}\sum_a m \boxed{F_{ab}\nabla_a W(r_{ab}, h_a)}$$

scatter contribution

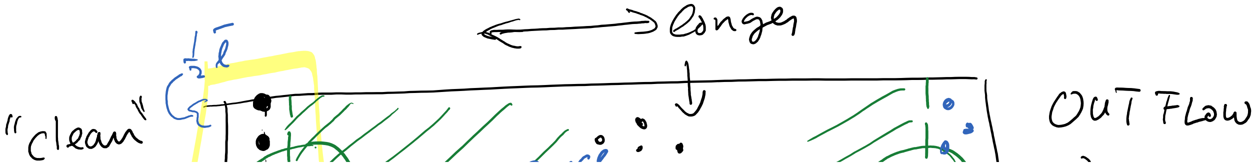


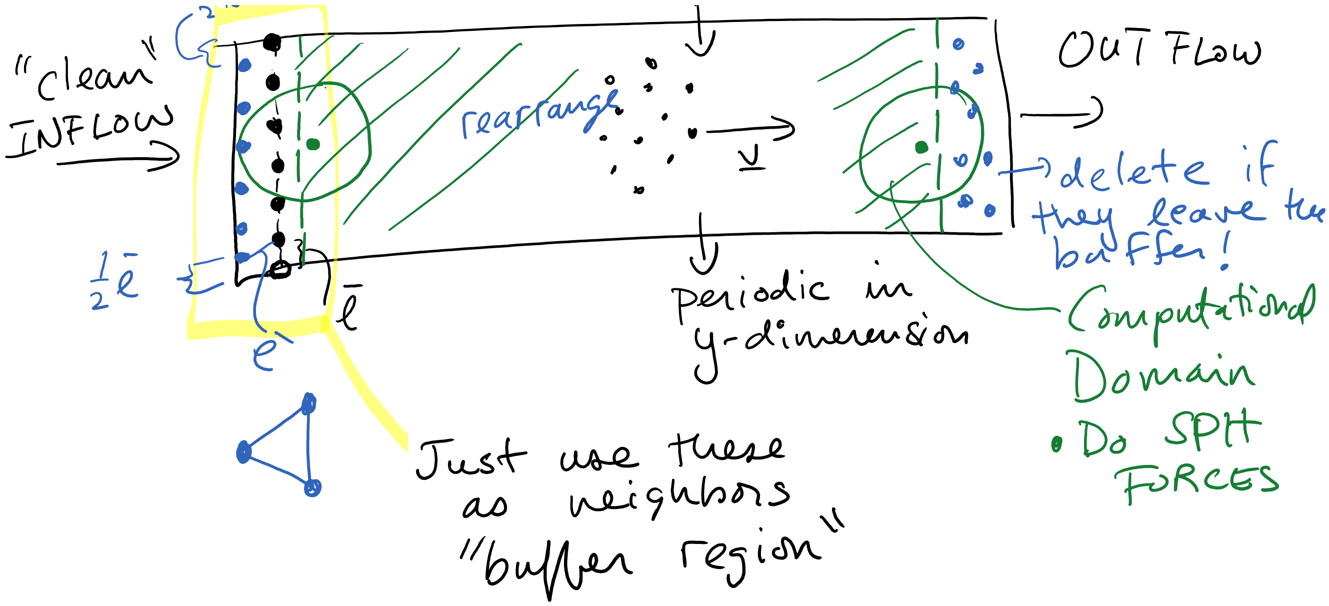
Only need to calculate the contribution in the $\square$ once for each neighbour.

Gather - Scatter Algorithm which conserves momentum.

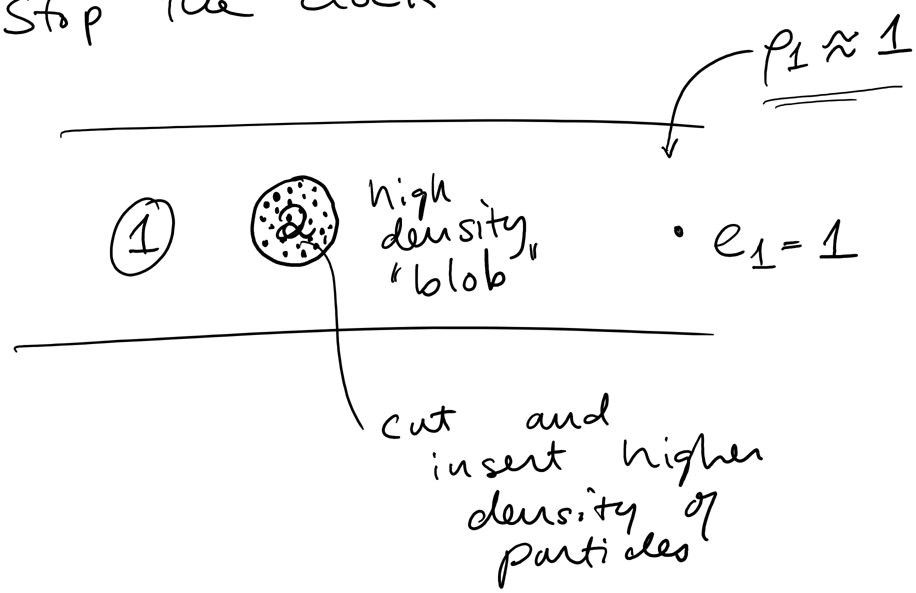
★ Remember to <u>initialize</u> $\left.\frac{d\underline{v}}{dt}\right|_a = \underline{0}$ for all particles.

★ Wind Tunnel:





☆ Let it run a bit, all OK?
Stop the clock



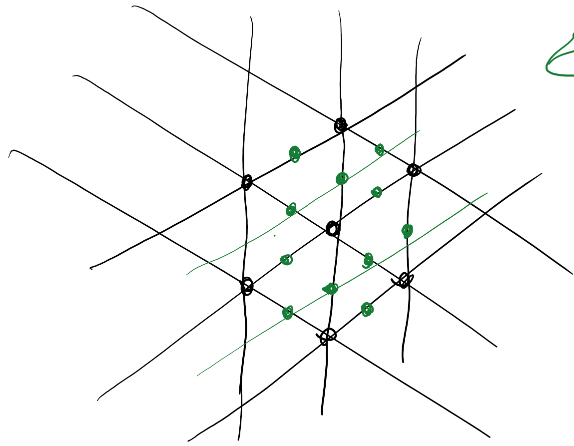
$$P_1 = P_2$$

$$e = \frac{P}{\rho(\gamma-1)}$$

$$\boxed{e_1 \rho_1 = e_2 \rho_2}$$

$$\rho_2 = 4 \cdot \rho_1$$

hence $e_2 = \frac{1}{4} e_1$



Code to generate these easily.