

$N$  cities

Gets very hard as the number of cities increases.

$\Theta(2^N)$  ,  $\Theta(N^k)$

NP-complete

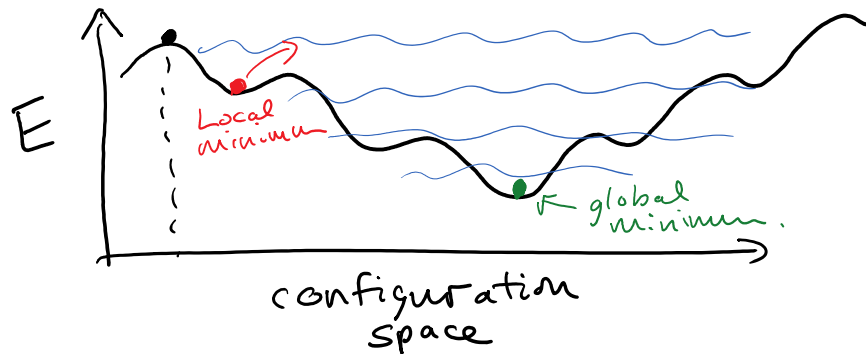
$P \equiv NP ?$

Optimize the path length

$$E = L_{\text{tour}} = \sum_{i=1}^N \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$$

$x_0 = x_N$   
 $y_0 = y_N$  to make it a cycle.

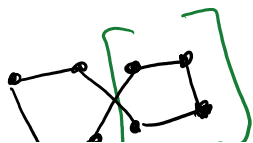
Want the global minimum Energy (or path length in this case).



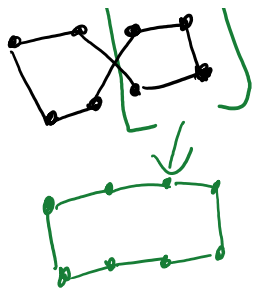
Moves: Ising  $\rightarrow$  Flip one random spin

- Swap 2 random cities ("random spin flip")

- Choose a random segment of the tour



$\rightarrow$  cut it out, this



Reverse  
the  
segment

cut it out this  
segment and  
reinsert it at a  
random  
position.

$$P(E, E', T) = \begin{cases} 1, & E' < E \\ \exp\left(\frac{E - E'}{T}\right), & E' \geq E \end{cases}$$

### ★ Metropolis Method

How to set initial  $T_0$ :

use 100 Random Moves and take  
the largest  $\Delta E \rightarrow$  make this our  
initial temperature ( $k_B = 1$ ).

Make  $O(1000)$  Moves at  $T_0$  keep the  
best (lowest  $E$ ) tour

$$T_{n+1} = 0.9 T_n$$

$C_i \rightarrow C_j$  in  $O(N^k)$  Moves  
any any

Euclidean Traveling Salesman problem is a  
special case which allows a lot of  
heuristics to improve the speed.

True optimum can be found for this case  
up to about 100'000 cities

time up to<sup>-1</sup> about 100'000 cities

TSPLIB

web link: <http://www.math.uwaterloo.ca/tsp/>

<http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/>