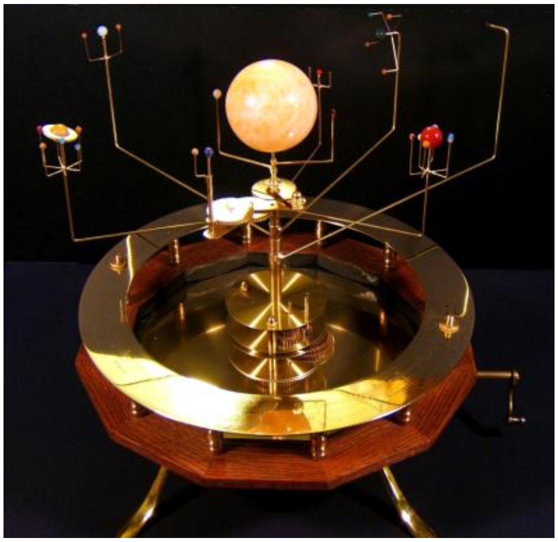
Many bodies: A digital Orrery

01 November 2021 12:21



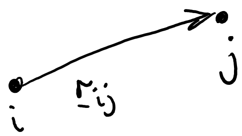
8 planets + Sun

9 bodies

$$H = T + U$$

Kinetic Energy

Potential Energy (9 bodies)



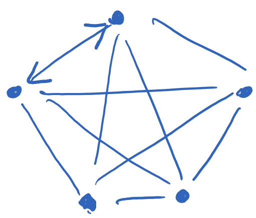
$$\underline{F}_{ij} = \frac{G m_i m_j}{|\underline{r}_j - \underline{r}_i|^2} \cdot \frac{(\underline{r}_j - \underline{r}_i)}{|\underline{r}_j - \underline{r}_i|}$$

$$\boxed{\underline{F}_{ij}} = \frac{G m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} \cdot (\underline{r}_j - \underline{r}_i) \quad \boxed{i \neq j}$$

Newton's Law of Gravity

$$\boxed{\underline{F}_{ij} = -\underline{F}_{ji}}$$

Newton's 3rd Law



$5 \times 4 = 20$ Forces

using Newton's 3rd Law we actually only have to calculate 10 Interactions.

$$N\text{-bodies} \Rightarrow \frac{N(N-1)}{2} \text{ Interactions to calculate}$$

$\underline{\mathcal{O}(N^2)}$ - Method

Imagine how many operations are needed to calculate 10^{12} bodies. Possible?

$\approx 10^{25}$ operations

"Exaflop" $= 10^{18}$ ops/s

10^{7} s $\Rightarrow$ 1/3 year

$\mathcal{O}(N \log N)$

$\mathcal{O}(N)$

Improved Methods to calculate Forces? Yes

$$G_N = 6.\underbrace{6742}\times 10^{-11}\ [m^3 kg^{-1} s^{-1}]$$

$$G_N \cdot M_\odot = k^2 \quad \text{Gauss' Grav. Constant}$$

$$k = 0.01720209895\ [AU^{3/2} M_\odot^{-1/2} D^{-1}]$$

$$\boxed{\underline{F}_i = \sum_{j \neq i} \frac{k^2 m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} (\underline{r}_j - \underline{r}_i)}$$

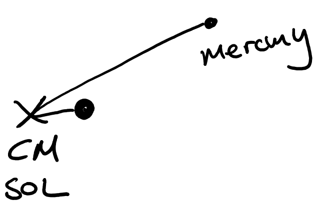
1D = 86400 S.I. seconds

$$-\frac{\partial H}{\partial q} = -\frac{\partial \phi(q)}{\partial q} \equiv -\nabla_q \phi$$

Acceleration

$$\boxed{\underline{a}_i = \frac{\underline{F}_i}{m_i}}$$

Barycentric Coordinates



$$\underline{a} = -\nabla \phi$$

Leapfrog

Drift "H = T" N $\boxed{\underline{r}_{\frac{1}{2},i}} = \underline{r}_{0,i} + \frac{h}{2} \underline{v}_{0,i}$

Leaping

Drift "H=T" N: $\underline{r}_{\frac{1}{2},i} = \underline{r}_{0,i} + \frac{h}{2}\underline{v}_{0,i}$

Kick "H=U" N: $\underline{v}_{1,i} = \underline{v}_{0,i} + h\,\underline{a}_i(\{\underline{r}_{\frac{1}{2}}\})$ $\quad \underline{\underline{O(N^2)}}$

"H=T" N: $\underline{r}_{1,i} = \underline{r}_{\frac{1}{2},i} + \frac{h}{2}\underline{v}_{1,i}$

Pseudocode?

$\underline{\Delta r} = \underline{r}_j - \underline{r}_i$ $\quad$ 3⊖, 3⊜

$|\;|^3$: $r2 = \Delta x * \Delta x + \Delta y * \Delta y + \Delta z * \Delta z$ $\quad$ 3⊛ 2⊕ 1⊜

$1/r$ $\quad ir = 1/\sqrt{r2}$ $\quad$ 1⊘ 1⊜

$ir3 = ir * ir * ir$

$mir3 = m_i * m_j * ir3$

$f_x = mir3 * \Delta x$

$f_y = mir3 * \Delta y$

$f_z = mir3 * \Delta z$

$\underline{a}[i] \mathrel{+}= \underline{f} * im[i]$ $\quad \frac{1}{m_i}$

$\boxed{\underline{a}[j] \mathrel{-}= \underline{f} * im[j]}$

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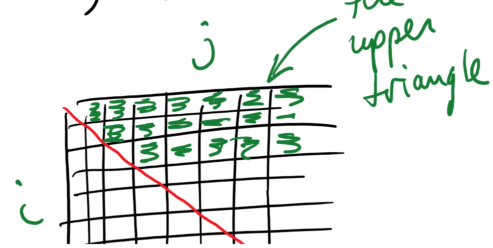
for (i=0; i<N; ++i)
    for (j=i+1; j<N; ++j) {
        execute the
        code above
    }
```



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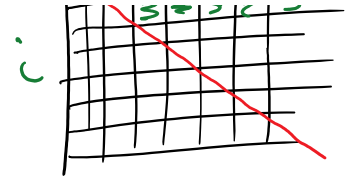

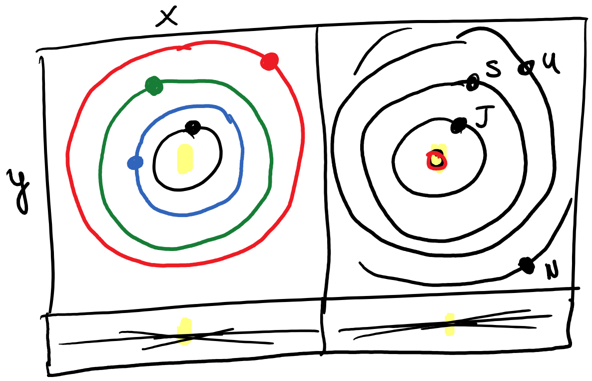

$\Delta t = 4$ days

We need some I.C.s in AU for $\underline{r}$

$\frac{AU}{day}$ for $\underline{v}$

m in $M_\odot$

Integrate to "today"