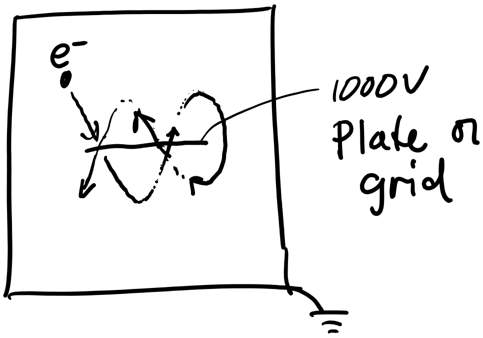
Bi-linear and Bi-cubic Interpolation: Electron Beams

Monday, 14 November 2022 12:28



potential

$$\underline{F} = m\,\underline{a}$$

electron

PDE (S.O.R.) ODE

Electrodynamics

$$\underline{F} = e(\underline{E} + \cancel{\underline{v}_e \times \underline{B}})$$

ignore

$$\underline{E} \rightarrow -\nabla\underline{\Phi}$$

$$\underline{a} = \left(\frac{e}{m_e}\right)\left(-\nabla\underline{\Phi}\right)$$

$$\dot{v}_x = \ddot{x} = \left(\frac{e}{m_e}\right)\left(-\frac{\partial\Phi}{\partial x}\right)$$

$$\dot{v}_y = \left(\frac{e}{m_e}\right)\left(-\frac{\partial\Phi}{\partial y}\right)$$

$$\dot{x} = v_x$$

$$\dot{y} = v_y$$

$$\ddot{x} \equiv \frac{d^2x}{dt^2}$$

System of 4 1st-order O.D.Es

Units

$$e = 1.6\times10^{-19}\,C$$

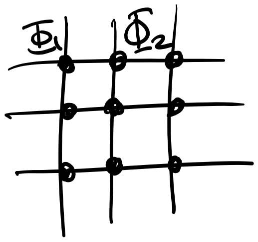
$$m_e = 9.11\times10^{-31}\,kg$$

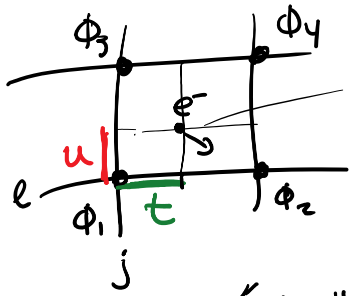
$$\left(\frac{e}{m_e}\right) = 1.76\times10^{11}\,C/kg$$

$$\Phi = \left[\frac{Nm}{C}\right] \qquad \nabla\Phi = \left[\frac{N\cancel{m}}{C\,\cancel{m}}\right]$$

$$\left(\frac{e}{m_e}\right)\left(-\nabla\Phi\right) = \left[\frac{C}{kg}\right]\cdot\left[\frac{N}{C}\right] = \frac{N}{kg} = \frac{\cancel{kg}\,ms^{-2}}{\cancel{kg}}$$

$= [m\, s^{-2}]$ ✓





$\phi, \frac{\partial \Phi}{\partial x}, \frac{\partial \Phi}{\partial y}$

Interpolation:

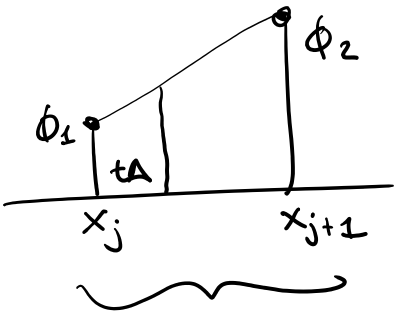
$t \in [0, 1]$

$u \in [0, 1]$

"Mix" these 4 potentials to estimate the value in the middle somewhere

Linear mixing

$$t = \frac{(x - x_j)}{\Delta} \qquad u = \frac{1}{\Delta}(y - y_e)$$



$$\phi = t\,\phi_2 + (1-t)\,\phi_1$$

if $t=0$ $\phi = \phi_1$

if $t=1$ $\Phi = \Phi_2$

$u=0 \Rightarrow \phi(x, y_e) = \boxed{(1-t)\,\phi_1 + t\,\phi_2}$

$u=1 \Rightarrow \phi(x, y_e) = \boxed{(1-t)\,\phi_3 + t\,\phi_4}$

Mix these 2 using u

$$\Phi(x,y) = (1-u)\left[(1-t)\phi_1 + t\phi_2\right]$$

$$u\left[(1-t)\phi_3 + t\phi_4\right]$$

$$\boxed{\Phi(x,y) = (1-u)(1-t)\,\phi_1 + (1-u)\,t\,\phi_2 + u(1-t)\,\phi_3 + u\,t\,\phi_4}$$

$$\Phi(x,y) = (1-u)(1-t)\,\Phi_1 + (1-u)\,t\,\Phi_2 + u(1-t)\,\Phi_3 + u\,t\,\Phi_4$$

2-D Bi-linear interpolation

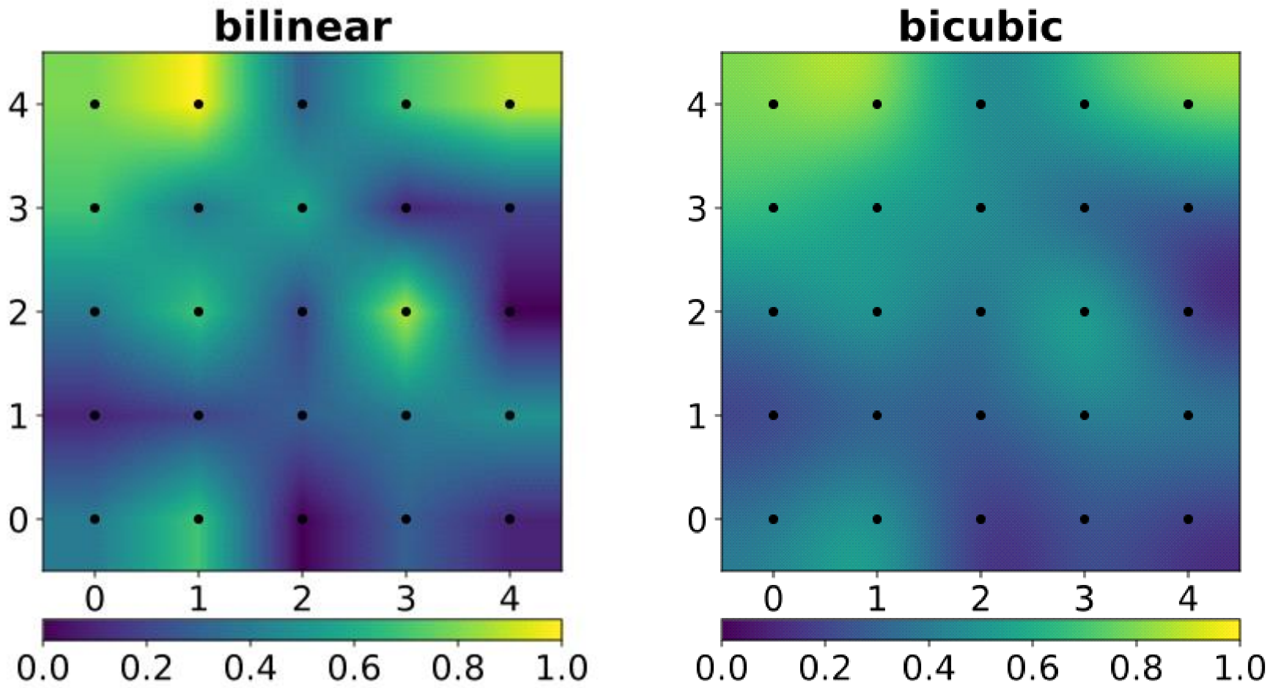
$$\left.\frac{\partial \Phi}{\partial x}\right|_u = \frac{\partial t}{\partial x}\cdot\frac{\partial \Phi}{\partial t} = \frac{1}{\Delta}\Big[-(1-u)\Phi_1 + (1-u)\Phi_2 - u\Phi_3 + u\Phi_4\Big]$$

$$= \frac{1}{\Delta}\Big[(1-u)(\Phi_2-\Phi_1) + u(\Phi_4-\Phi_3)\Big]$$

$$\begin{cases} \dot{x} = v_x \\ \dot{v}_x = \left(\frac{e}{m_e}\right)\Big[-\frac{1}{\Delta}\Big((1-u)(\Phi_2-\Phi_1) + u(\Phi_4-\Phi_3)\Big)\Big] \\ \dot{y} = v_y \\ \dot{v}_y = \left(\frac{e}{m_e}\right)\Big[-\frac{1}{\Delta}\Big(\cdots\cdots\Big)\Big] \end{cases}$$

exercise

RK4 or Leapfrog method.



Bicubic:

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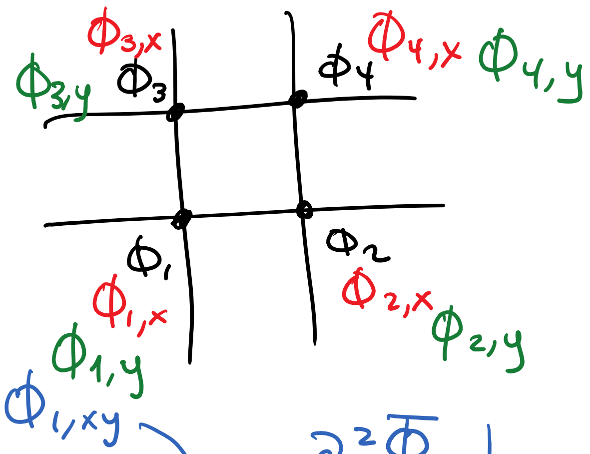
$$\Phi(t,u) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}\, t^i u^j$$

$$\begin{bmatrix} a_{00} & a_{01} & \cdots & \\ a_{10} & a_{11} & & \\ \vdots & & a_{22} & \\ & & & a_{33} \end{bmatrix} = M^T \cdot \left[\begin{array}{cc|cc} \Phi_1 & \Phi_2 & \Phi_{1,y} & \Phi_{2,y} \\ \Phi_3 & \Phi_4 & \Phi_{3,y} & \Phi_{4,y} \\ \hline \Phi_{1,x} & \Phi_{2,x} & \Phi_{1,xy} & \Phi_{2,xy} \\ \Phi_{3,x} & \Phi_{4,x} & \Phi_{3,xy} & \Phi_{4,xy} \end{array}\right] \cdot M$$

$$M = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 0 & 3 & -2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$



$$\left.\frac{\partial^2 \Phi}{\partial x \partial y}\right|_{\substack{t=0\\u=0}}$$

$$\Phi(t,u) = \begin{bmatrix} 1 & t & t^2 & t^3 \end{bmatrix} \cdot \begin{bmatrix} a_{ij} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ u \\ u^2 \\ u^3 \end{bmatrix}$$

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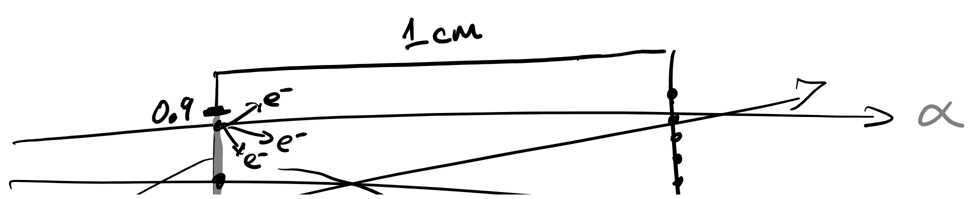
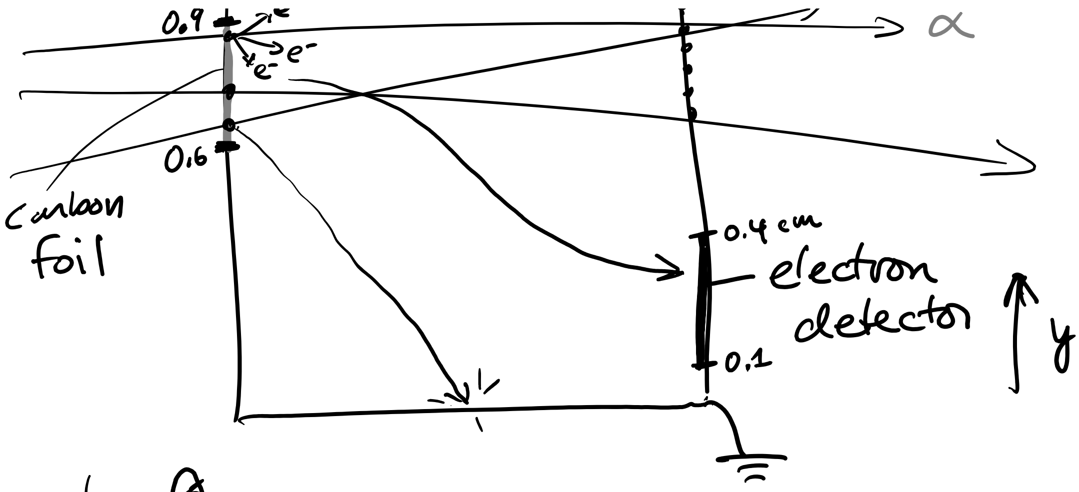
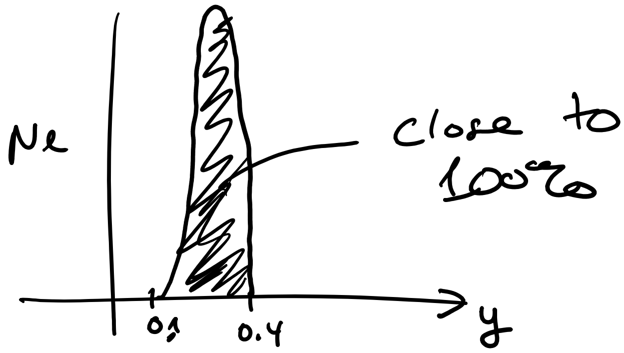
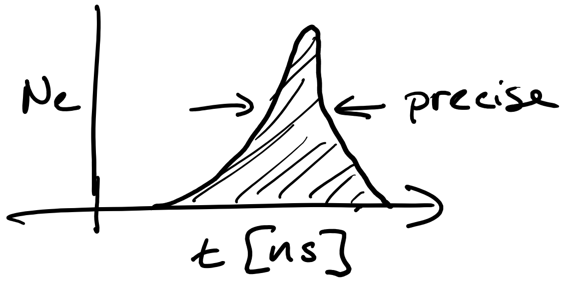
2 Weeks
- Week 1 : VALIDATION
- Week 2 : DESIGNING









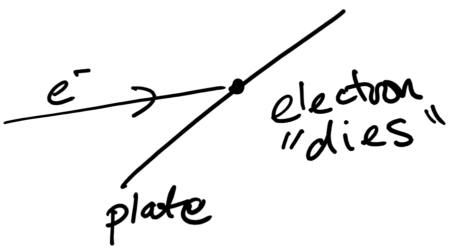
Initial electron velocity

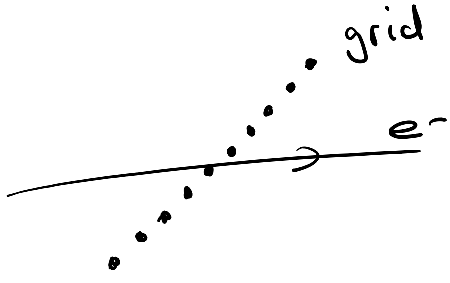
$$|v_e| = 10^6 \ m/s$$

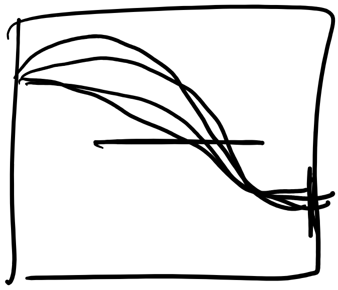
at random angles

Draw Boundary conditions

$\rightarrow \Phi , R$







Minimize the number of elements