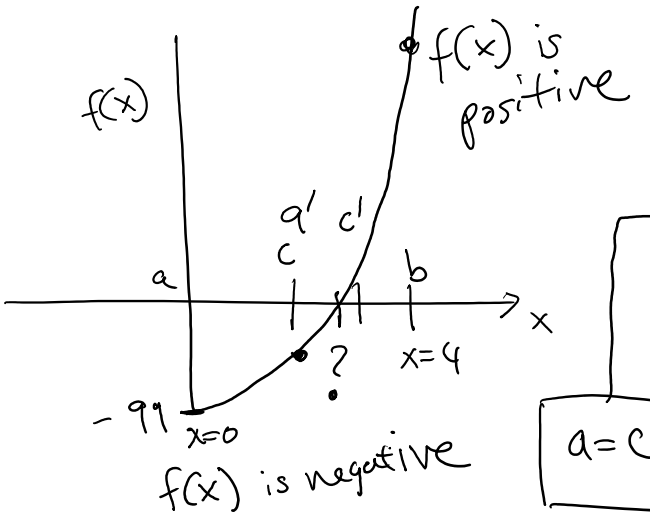
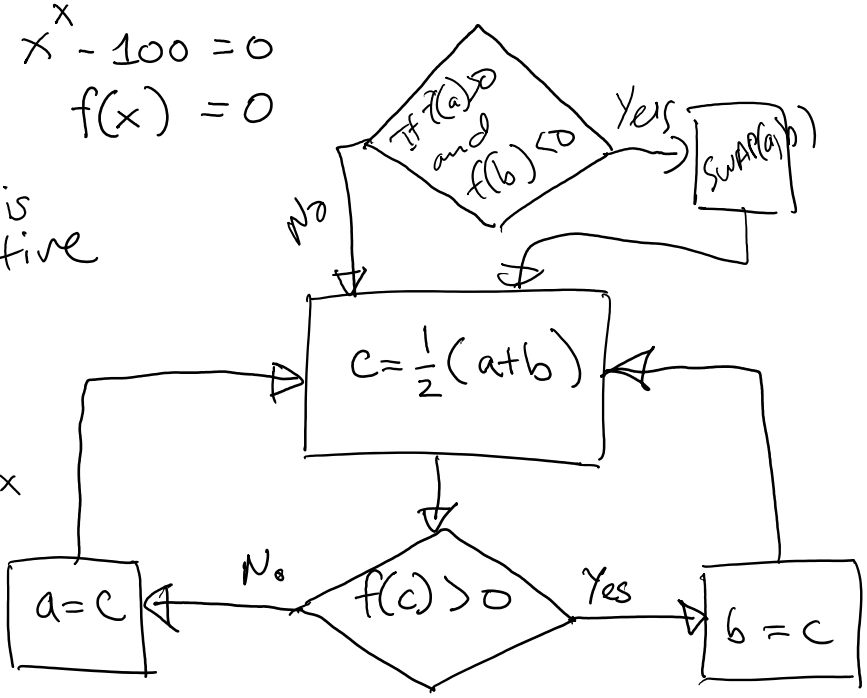
SINS 1: Newton Method and Kepler's Equation

16 September 2019 12:52

$x^x = 100 \Rightarrow x^x - 100 = 0$

$f(x) = 0$





1. Infite - never terminates!
2. f(x) could be decreasing

If at the start f(a) positive and f(b) is negative then SWAP(a, b)

3. Search for a bracket of the root first!

$|a-b| < \varepsilon_{ABSOLUTE}$

$\frac{|a-b|}{|c|} < \varepsilon_{RELATIVE}$ 0.1%

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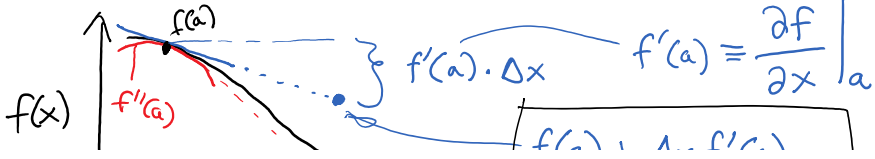
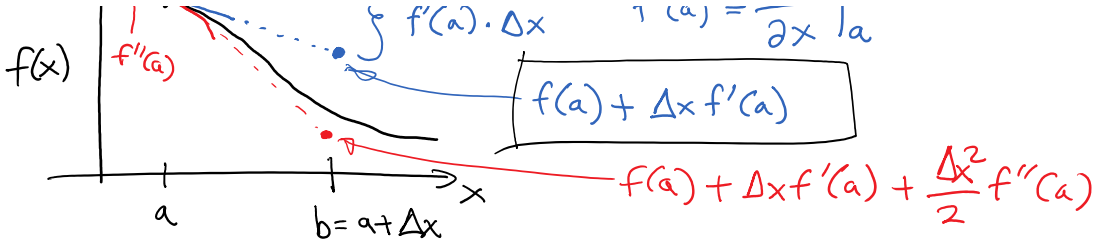
* and + $\sqrt{x}$, sin(x)

The Taylor Expansion





$$f(b) \equiv f(a+\Delta x) \approx f(a) + \Delta x f'(a) + \frac{1}{2}\Delta x^2 f''(a) + \frac{1}{6}\Delta x^3 f^{(3)}(a) + \cdots + \frac{1}{n!}\Delta x^n f^{(n)}(a) + \Theta(\Delta x^{n+1})$$

$$\left[\frac{\Delta x^n}{(n-1)!}\int_0^1 (1-t)^{n-1} f^{(n)}(a+t\Delta x)\,dt\right]$$

Ignore all terms of order 2 and higher!

$$f(a+\Delta x) \approx f(a) + \Delta x f'(a)$$

Suppose our function has a root at x

$$f(x) = 0 \qquad x \equiv a + \Delta x$$

$$0 \cong f(a) + \Delta x \cdot f'(a)$$

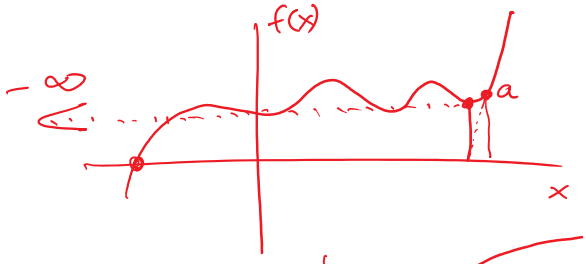
$$\Delta x \cong -\frac{f(a)}{f'(a)}$$

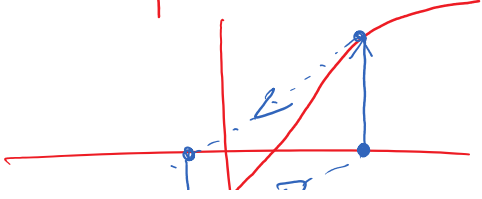
$$x \cong a - \frac{f(a)}{f'(a)}$$

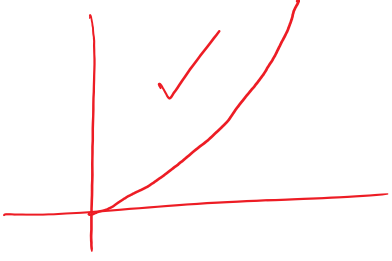
Newton's Method

$a = x$

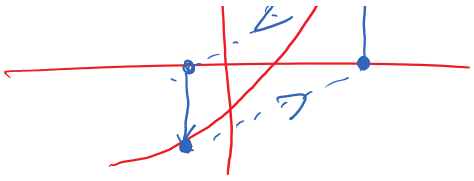
Quadratically Converging







Cyclical behaviors



Cyclical behaviour

Example: $\sqrt{x} \rightarrow \left(\frac{1}{\sqrt{x}}\right) \cdot x$ ← Is Given!

$y = \frac{1}{\sqrt{x}}$ $\quad f(y) = y - \frac{1}{\sqrt{x}} = 0$

No! Don't solve this: $y^2 = \frac{1}{x}$

$$f(y) = x - \frac{1}{y^2}$$

$$y \leftarrow y - \frac{x - \frac{1}{y^2}}{2\frac{1}{y^3}}$$

$$y \leftarrow y - \frac{1}{2}xy^3 + \frac{1}{2}y$$

$$y \leftarrow y*(1.5 - 0.5**x*y*y)$$

$$x = +M \times 2^e \qquad 1 \le M < 2$$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{M}} \times 2^{-e/2}$$

$$\frac{\sqrt{2}}{2} < y < 1$$

Midpoint of this range is $\frac{1}{2}\left(1 + \frac{\sqrt{2}}{2}\right)$

≈ 0.85355

Kepler's Equation

$$M = E - e \sin E$$

M, E are $[0, 2\pi)$

M: known (mean)

e: known (eccentricity)

anomaly eccentricity

$$f(E) = E - e\sin E - M$$
$$f'(E) = 1 - e\cos E$$

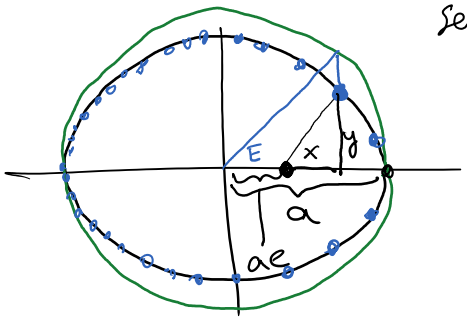
$$E_0 = M$$

$$M = nt \qquad n = \frac{2\pi}{T}$$

T – period

$$T = a^{3/2}$$

T: years, a: AU



semi-major axis → $x = a(\cos E - e)$

$$y = a\sqrt{1-e^2}\sin E$$

b "semi-minor axis"

$$\Delta t = \frac{1}{52}$$ 88

Note: $M = \left(\frac{t}{T}\right)\cdot 2\pi$