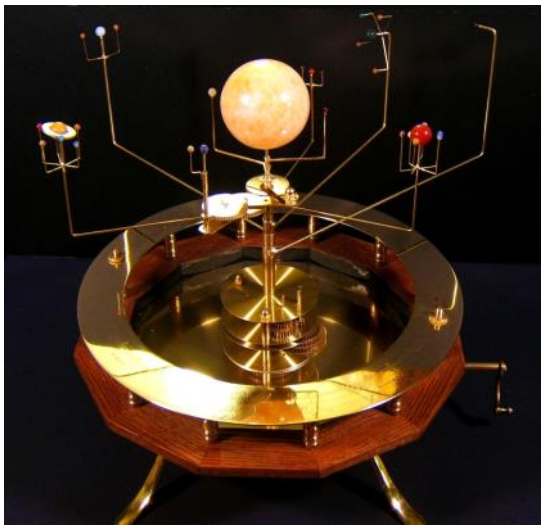
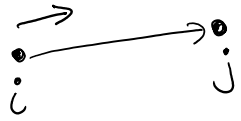
Gravitation for many bodies: The Solar System Orrery

14 October 2019 12:49



8 planets and the Sun

9 Bodies

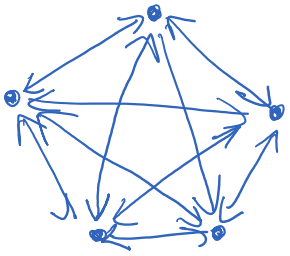


$$\underline{F}_{ij} = \frac{G m_i m_j}{|\underline{r}_j - \underline{r}_i|^2} \frac{(\underline{r}_j - \underline{r}_i)}{|\underline{r}_j - \underline{r}_i|}$$

$$\underline{F}_{ij} = \frac{G m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} (\underline{r}_j - \underline{r}_i) \qquad i \neq j$$

$$\boxed{\underline{F}_{ij} = -\underline{F}_{ji}}$$ Newton's 3rd Law

5 bodies



$5 \times 4 = 20$

$$\frac{N \cdot (N-1)}{2}$$ Interactions

$$\frac{9 \cdot 8}{2} = 36$$

$O(N^2)$ order of the algorithm

$O(N \log N)$

$O(N)$

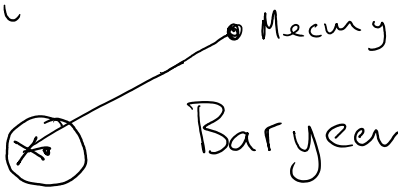
$$G_N = 6.6742 \times 10^{-11} \left[m^3 kg^{-1} s^{-2} \right]$$

$$k = 0.01720209895 \left[AU^{3/2} M_\odot^{-1/2} D^{-1} \right]$$

$$\underline{F}_i = \sum_{j \neq i} \frac{k^2 m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} (\underline{r}_j - \underline{r}_i)$$

1 D = 86400 S.I. seconds!

$$\underline{a}_i = \frac{\underline{F}_i}{m_i}$$



Baryecenter Coordinates

$O(N)$ $\underline{r}_{\frac{1}{2},i} = \underline{r}_{0,i} + \frac{h}{2} \underline{v}_{0,i}$ 9x 27 variables

$O(N^2)$ $\underline{v}_{1,i} = \underline{v}_{0,i} + h \underline{a}_i(\{\underline{r}_{\frac{1}{2}}\})$

$O(N)$ $\underline{r}_{1,i} = \underline{r}_{\frac{1}{2},i} + \frac{h}{2} \underline{v}_{1,i}$

1. $\Delta \underline{r} = \underline{r}_j - \underline{r}_i$ 3⊖

$r2 = \Delta x * \Delta x + \Delta y * \Delta y + \Delta z * \Delta z$

$ir = \frac{1}{\sqrt{r2}}$

$ir3 = ir * ir * ir \left(\frac{1}{|\underline{r}_j - \underline{r}_i|^3} \right)$

```
mir3 = mi * mj * ir3

f_ { fx = mir3 * Δx
   { fy = mir3 * Δy
   { fz = mir3 * Δz

a[i] += f * im[i]      <- 1/mi
a[j] -= f * im[j]
```

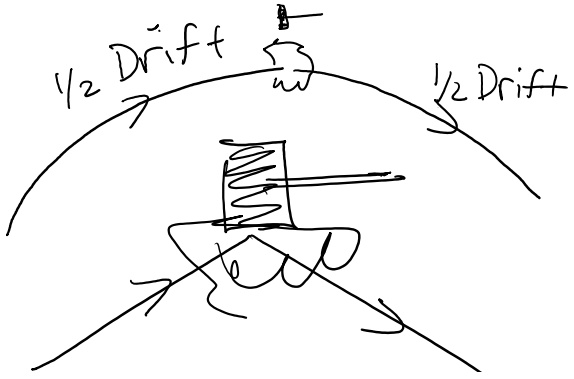
```
for (i = 0; i < N; ++i) {
    for (j = i+1; j < N; ++j) {
        //// that code above ////
    }
}
```

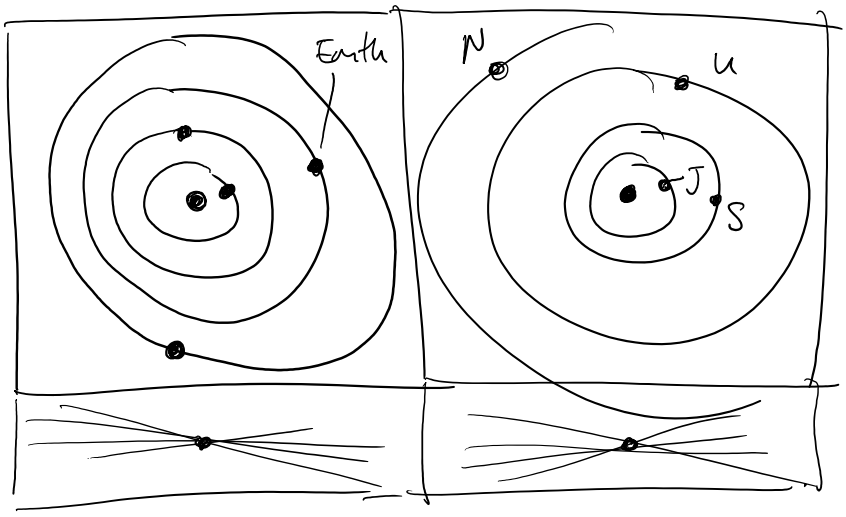
"STATE-OF-THE-ART"

$$H = T + U$$

$$H = H_{KEPLER} + H_{PERTURB}$$

SUN-PLANET PLANET-PLANET




Earth
N
U
J
S