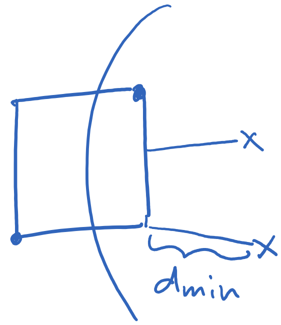
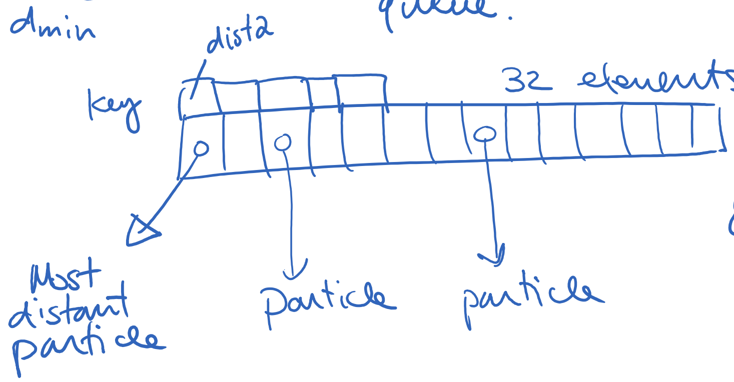
Gravity Tree Walk

02 March 2020 11:57



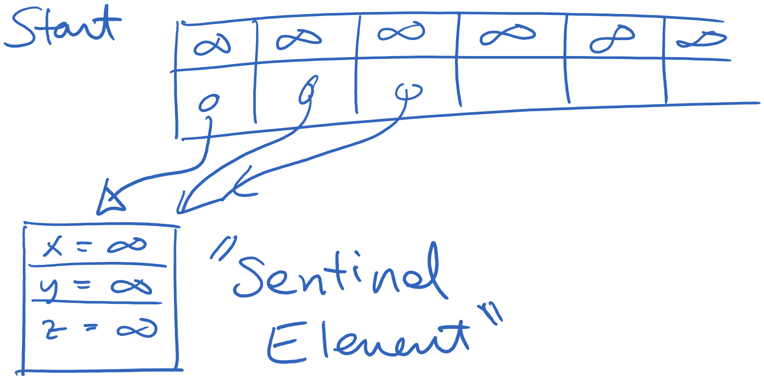
PRIORITY QUEUE

- keeps track of the most distant particle in the queue.

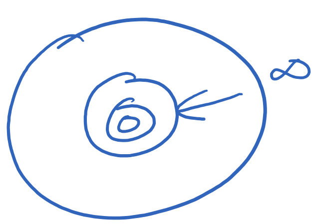


$O(N)$

Start



"Sentinel Element"



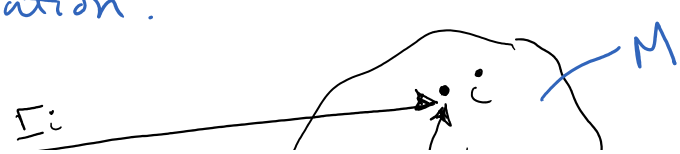
Shrinking Ball Search

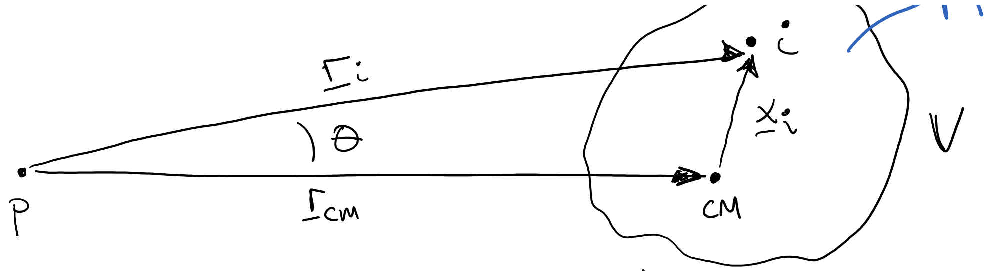
N - 9 bodies Gravity 36 pairwise interactions "Forces"

$O(N^2)$

Tree code $O(N \log N)$: uses a tree

☆ The tree is a hierarchical description of the mass distribution.





$$|\underline{x}_i| \ll |\underline{r}_{cm}|$$

$$\Psi_P = -\sum_{i\in V} \frac{\overset{=1}{G}\, m_i}{|\underline{r}_i|} = \sum_{i\in V} m_i\, \gamma(|\underline{r}_i|) \qquad \gamma \equiv -\frac{1}{r}$$

$$= \sum_{i\in V} m_i\, \gamma\left(|\underline{r}_{cm} + \underline{x}_i|\right)$$

Vector index

$$= \sum_{i\in V} m_i \Big[\gamma(r_{cm}) + \frac{\partial}{\partial r^j}\gamma(r_{cm})\, x_i^j$$

particle index

$$+ \frac{1}{2} \underbrace{\frac{\partial^2}{\partial r^j \partial r^k}\gamma(r_{cm})}\cdot \overbrace{x_i^{jk}} + \cdots \Big]$$

also Rank-2 Tensor

$$\begin{bmatrix} xx & xy & xz \\ yx & yy & yz \\ zx & zy & zz \end{bmatrix}_i$$

Now pull the sum into the expression so that it operates only on the i-dependent factors.

$$\Psi = \gamma(r_{cm}) \cdot \underbrace{\sum_i m_i}_{\underline{M}\ \text{monopole}} + \frac{\partial}{\partial r^j}\gamma(r_{cm}) \cdot \underbrace{\sum_i m_i x_i^j}_{\underline{M}^j\ \text{diapole}}$$

$$\frac{1}{2}\frac{\partial^2}{\ } \gamma(\qquad \sum m_i x_i^{jk}$$

monopole dipole

$$+\frac{1}{2}\frac{\partial^2}{\partial r^j \partial r^k}\gamma(r_{cm})\cdot\underbrace{\sum_i m_i x_i^{jk}}_{M^{jk}}$$

Now what about the derivatives?

$$\boxed{\begin{aligned}&\gamma(r) \equiv \gamma_0 = -\frac{1}{r} \qquad \gamma_{m+1} = -\frac{(2m+1)}{r^2}\gamma_m \\ &\frac{\partial}{\partial r}\gamma_m = \gamma_{m+1}\,\underline{r}\end{aligned}}$$

$$\gamma_0 = -\frac{1}{r} \qquad \gamma_1 = -\frac{1}{r^2}\gamma_0 = \frac{1}{r^3}$$

$$\gamma_2 = -\frac{3}{r^2}\cdot\frac{1}{r^3} = -\frac{3}{r^5}$$

$$\gamma_3 = -\frac{5}{r^2}\cdot\left(-\frac{3}{r^5}\right) = \frac{15}{r^7}$$

$$\partial \equiv \frac{\partial}{\partial r} \qquad \partial_j \equiv \frac{\partial}{\partial r^j}$$ Notation

$$\partial\gamma_0 = \gamma_1\,\underline{r} = \frac{\underline{r}}{r^3}$$

$$\partial_{jk}\gamma_0 = \partial_j(\gamma_1 r^k) = (\gamma_2 r^{jk} + \gamma_1 \delta^{jk})$$

$M^j = 0$! about the center of mass

$r \equiv r_{cm}$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\Psi = -\frac{M}{r} + \frac{1}{2}\left(\gamma_2 r_{jk} + \underbrace{\gamma_1 \delta_{jk}) M^{jk}}_{Tr(M^{jk})}$$

↑ Quadrupole Moment

$$\boxed{\Psi_p = -\frac{M}{r} - \frac{3}{2r^5} r_{jk} M^{jk} + \frac{1}{2}\frac{1}{r^3} Tr(M^{jk})}$$

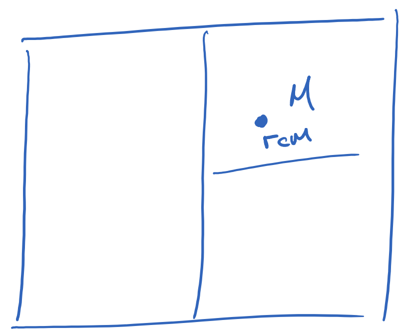
$$\boxed{\Psi_p = -\frac{M}{r} - \frac{\ldots}{2r^5} \ldots + \frac{\ldots}{2r^3} \ldots}$$

$a^l \equiv -\partial^l \Psi$ Need one more derivative

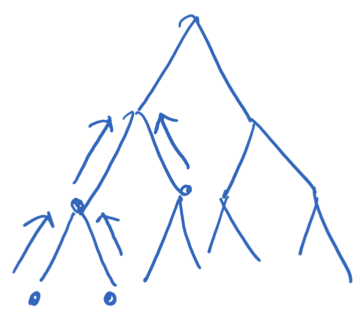
$$a^l = -\frac{M r^l}{r^3}$$

M_1, M_2

$\underline{r}_1, \underline{r}_2$

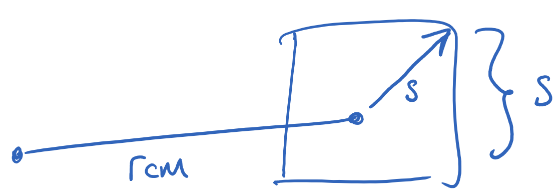


$$\underline{r} = \frac{M_1}{M}\underline{r}_1 + \frac{M_2}{M} r_2$$



One single upward pass to calculate M, $\underline{r}_{cm}$ for all cells in the tree.

$O(N)$



$$\frac{s}{r_{cm}} < \theta$$

if $r_{cm} < \frac{s}{\theta}$ then "open" the cell (recurse deeper)

else Calculate

$$\underline{a} \mathrel{+}= \frac{M \underline{r}}{r^3}$$

$\theta \approx 0.5$