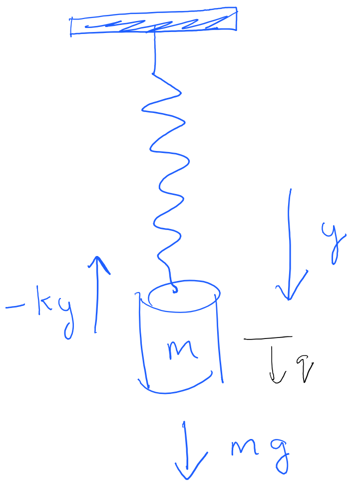
Harmonischer Oszillator



$$F = mg - ky$$

Noch einfacher $m=1$ $k=1$

$$F = g - y$$

Mit diesen Feder ist die Kraft der Feder $= 0$ bei $y=0$!

$$F = 0 \longrightarrow y = g$$

$$q = y - g = -F$$

$$\dot{q} = \dot{y}$$

$$\ddot{q} = -q$$

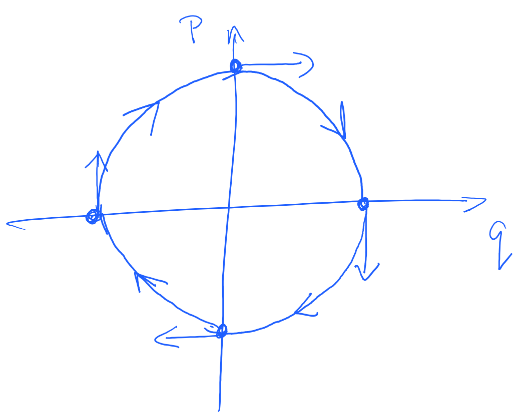
$$\ddot{q} = \ddot{y} = F = -q$$

$m=1$!

Momentum: $p = mv = m\dot{y} = \frac{dq}{dt}$

$$\boxed{\begin{aligned}\dot{q} &= p \\ \dot{p} &= -q\end{aligned}}$$

2 ODEs

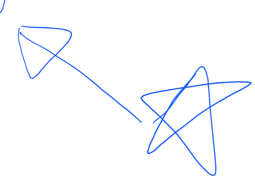


$$r^2 = q^2 + p^2$$

Erhalten während der Dynamik

$$H = \frac{1}{2}(q^2 + p^2)$$

Allgemein $\boxed{\dot{p} = -\frac{\partial H}{\partial q} \qquad \dot{q} = \frac{\partial H}{\partial p}}$



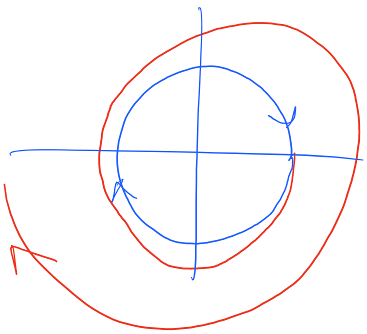
$$H = \underbrace{T(p)} + \underbrace{U(q)}$$

Kinetische Energie   Potenzielle Energie

Vorwärts-Euler:

$$q_{n+1} = q_n + h p_n \,, \quad p_{n+1} = p_n - h q_n$$

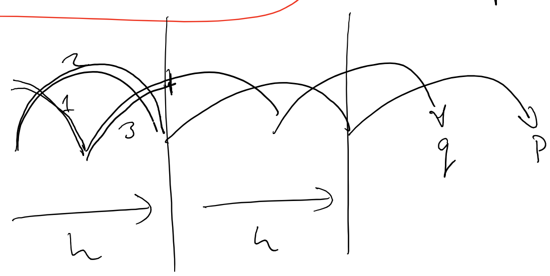
$$\Rightarrow \underbrace{q_{n+1}^2 + p_{n+1}^2}_{\text{Radius}} = (1+h^2)\underbrace{(q_n^2 + p_n^2)}_{\text{Radius}}$$



! BAD.

Symplektische Integratoren

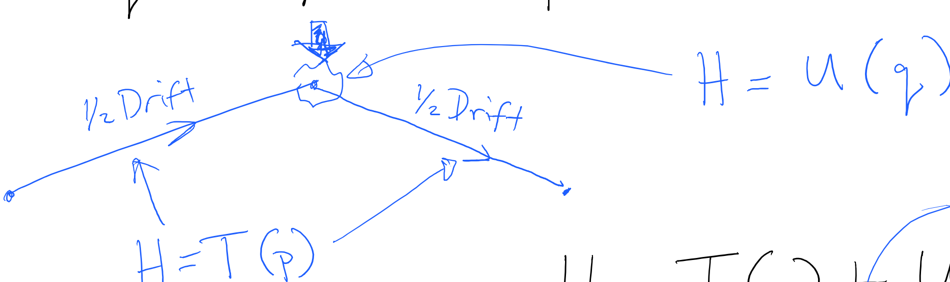
Leap-Frog o. Strömer-Verlet



$$q_{1/2} = q_0 + \tfrac{1}{2} h p_0 \quad \text{Drift}$$

$$p_1 = p_0 - h q_{1/2} \quad \text{Kick}$$

$$q_1 = q_{1/2} + \tfrac{1}{2} h p_1 \quad \text{Drift}$$



$$H = \underset{\frac{1}{2}mv^2}{\underline{T(p)}} + U(q)$$

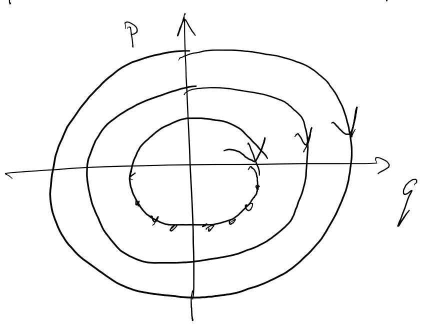
$$\underline{x}_{1/2} = \underline{x}_0 + \frac{1}{2} h \underline{v}_0$$

$$\underline{F} = -\nabla U$$

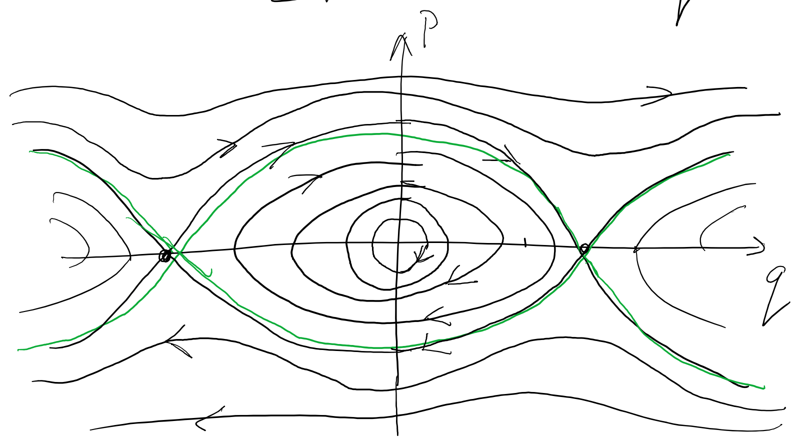
$$\underline{v}_1 = \underline{v}_0 + h\left(-\nabla U(\underline{x})\right)$$

$$\underline{x}_1 = \underline{x}_{1/2} + \frac{1}{2} h \underline{v}_1$$

H.O. $H = \frac{1}{2}(p^2 + q^2)$



$H = \frac{1}{2}p^2 - \varepsilon \cos q$



Numerischen Hamiltonian

$$\tilde{H} \cong H$$

Schritte kontinuierlich