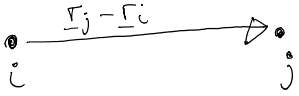
Spin 1 - Gravitation und Leap-frog: das Sonnensystem
Integrieren

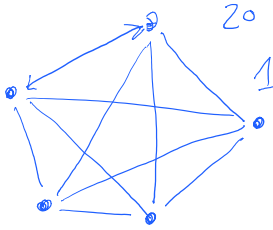
Monday, March 21, 2016 1:00 PM

8 Planeten und die Sonne



$$\underline{F}_{ij} = \frac{G m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} (\underline{r}_j - \underline{r}_i)$$

Vektor



20 Kräfte
10 Wechselwirkungen

$$= \frac{G m_i m_j}{|r_j - r_i|^2} \left(\frac{\underline{r}_j - \underline{r}_i}{|\underline{r}_j - \underline{r}_i|} \right)$$

Richtung!

$$\underline{F}_{ij} = -\underline{F}_{ji} \quad (\text{Newton 3})$$

Wir müssen eigentlich nur 10 Kräfte rechnen.

N-Körper

$\frac{N(N-1)}{2}$ - Kräfte $O(N^2)$

$$G_N = 6.6742 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$$

$$k = 0.01720209895 \left[AU^{3/2}\ M_\odot^{-1/2}\ D^{-1} \right]$$

$$1\,D\ (\text{Tag}) = 86400\ SI\ \text{Sekunden}$$

$$\underline{F}_i = \sum_j \frac{k^2 m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} (\underline{r}_j - \underline{r}_i) \quad \leftarrow O(N^2)$$

(r^3)

$(\underline{r}_j - \underline{r}_i)$ ... $d\,[k]$

$$\underline{a}_i = \underline{F}_i / m_i \quad (\text{Newton 1}) \quad \leftarrow O(N)$$

Alle Koordinaten müssen im Baryzentischen System sein.

$$x_{1/2} = x_0 + \frac{h}{2} v_0$$

$$v_1 = v_0 + h\, a(x_{1/2})$$

Alle Planeten und die Sonne

$v_{1/2} = v_0 + \frac{h}{2} a(x_0)$ Sonne

$x_1 = x_0 + h\, v_{1/2}$

Für Alle

$\forall i : \underline{r}_i \mathrel{+}= 0.5 * h * \underline{v}_i$

```
for (i=0; i<N; ++i) for (k=0; k<3; ++k)
                      planet[i].F[k]=0;
for (i=0; i<N-1; ++i) {

    for (j=i+1; j<N; ++j) {
        d2=0;
        for (k=0; k<3; ++k) {
            d[k] = planet[j].r[k] -
                   planet[i].r[k];
            d2 += d[k]*d[k];
        }
        ir = 1.0/sqrt(d2);   rsqrt(d2)
        ir3 = ir*ir*ir;
        ir3 *= planet[i].mass *
               planet[j].mass;
        ir3 *= kSq;
        for (k=0; k<3; ++k) {
            planet[i].F[k] += ir3*d[k];
            planet[j].F[k] -= ir3*d[k];
        }
    } /* end of j-loop */
} /* end of i-loop */
/* Now the Kick */
```

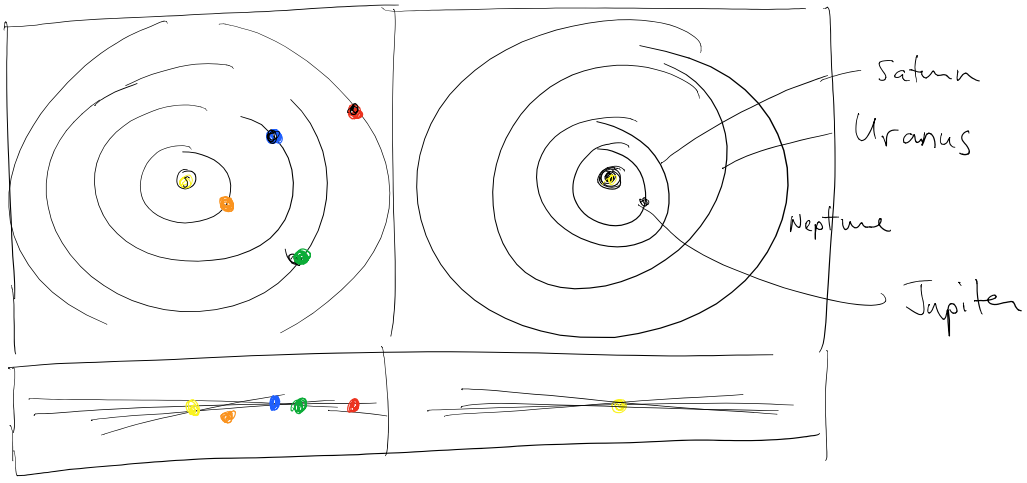
$\forall i : \underline{v}_i \mathrel{+}= h * (\underline{F}_i / m_i)$

$\forall i : \underline{r}_i \mathrel{+}= 0.5 * h * \underline{v}_i$

Was sollte h sein?

Leap-frog Stabilität Schritte/orbit > 20

$h < 4$ Tage !



$H = T + U$

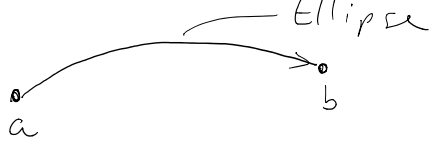
$H = H_{Kepler} + H_{Planet-Planet}$

Keine Sonne!

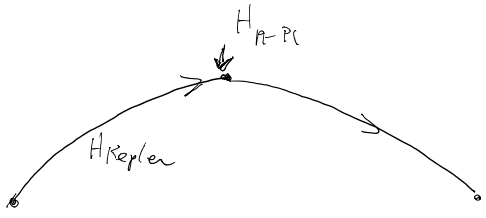
$O\left(\frac{M_{Jupiter}}{M_{Sonne}}\right) \sim 10^{-3}$

$H = T$ freies Teilchen

$H = H_{Kepler}$ "freies Teilchen"



Standard



1000x Genauer