Spin 1 - Bevölkerungswachstum, Chaos und Fraktale

Monday, April 4, 2016 12:28 PM

Nehmen wir an wir haben eine Bevölkerung mit Konstante Wachstumsrate r.

$$\frac{P_{n+1} - P_n}{P_n} = r$$

oder

$$P_{n+1} = (1+r) P_n$$

Es gibt eine Lösung für alle Zeit

$$P_{n+1} = (1+r)^n P_0 .$$

Unbeschränktes Wachstum.

$$\frac{1}{P}\frac{dP}{dt} = r$$

$$\frac{d \ln P}{dt} = r$$

$$\int_{\ln P_0}^{\ln P} d \ln P = \int_0^t r \, dt$$

$$\ln \frac{P}{P_0} = rt$$

$$P = P_0 e^{rt}$$

Normalizieren wir die Bevölkerung mit der "Maximalen" Grösse der Bevölkerung, N.

$$p = P/N$$

p	r ← nicht konstant!
1	0
~ 1	klein
klein	positiv, gross
>1	negativ - Aussterben!

VERHULST

$$r \propto \underbrace{(1 - p_n)}$$

$$\frac{p_{n+1} - p_n}{p_n} = r(1 - p_n)$$

$$\frac{dp}{dt} = rp(1-p)$$

$$p_{n+1} = p_n + r p_n (1 - p_n)$$

Quadratische Abhängigkeit

Nicht Linear!

Schon mit diesem einfachen Model gibt keine algemeine Lösung für alle Zeit.

Es ist deterministisch

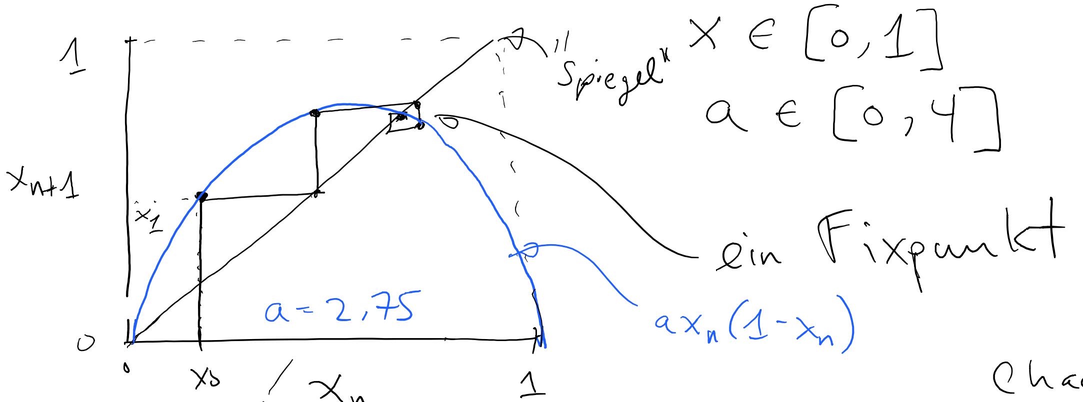
Eg. $r = 3$ $\quad p_0 = 0.01$

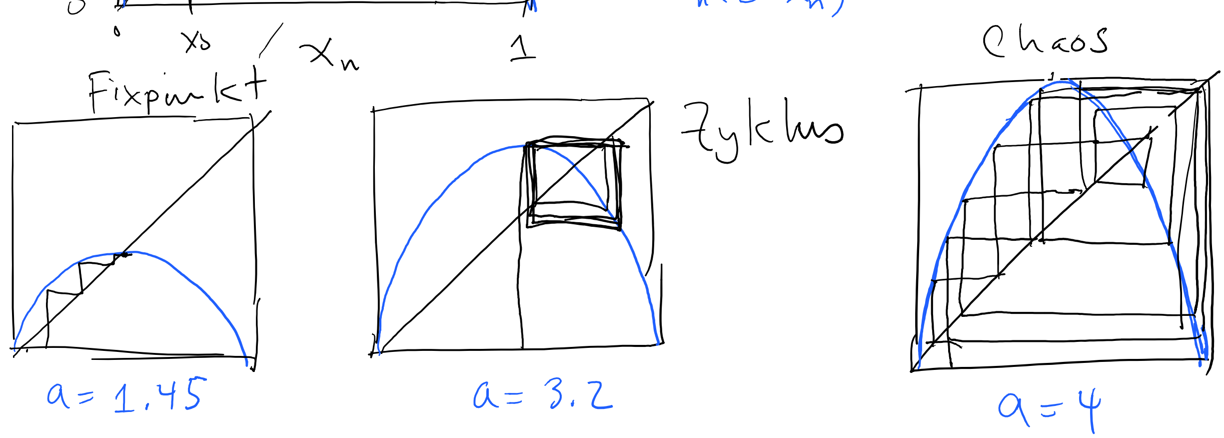
$p_0' = 0.00999999999\ldots$

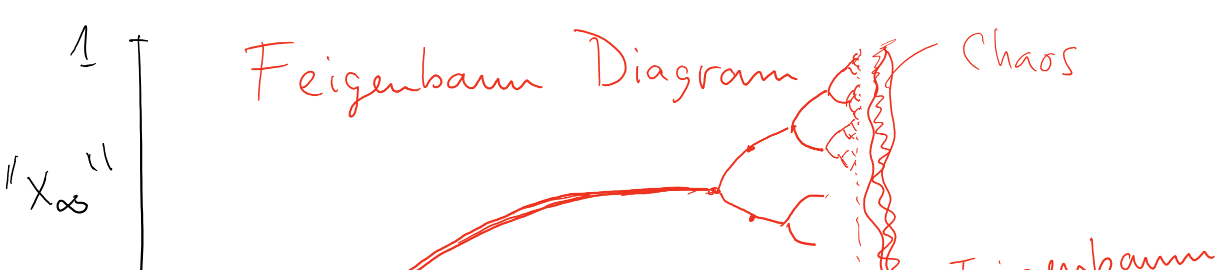
$$x_{n+1} = a\, x_n (1 - x_n)$$ Logistische Gleichg.

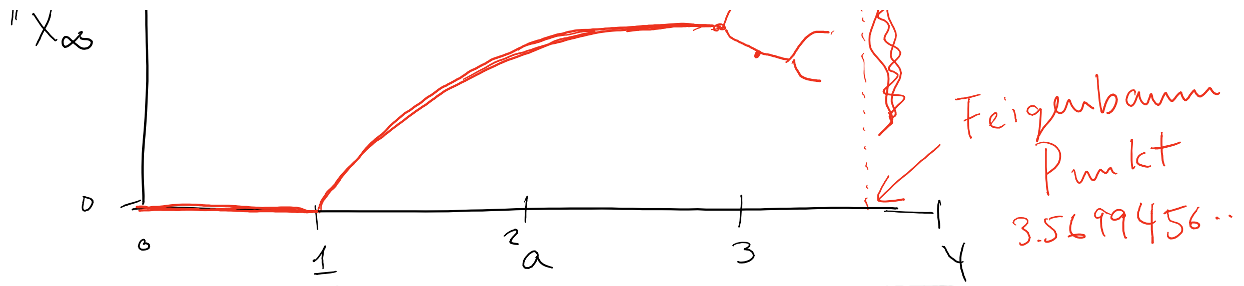
$x \in [0,1]$

$a \in [0,4]$

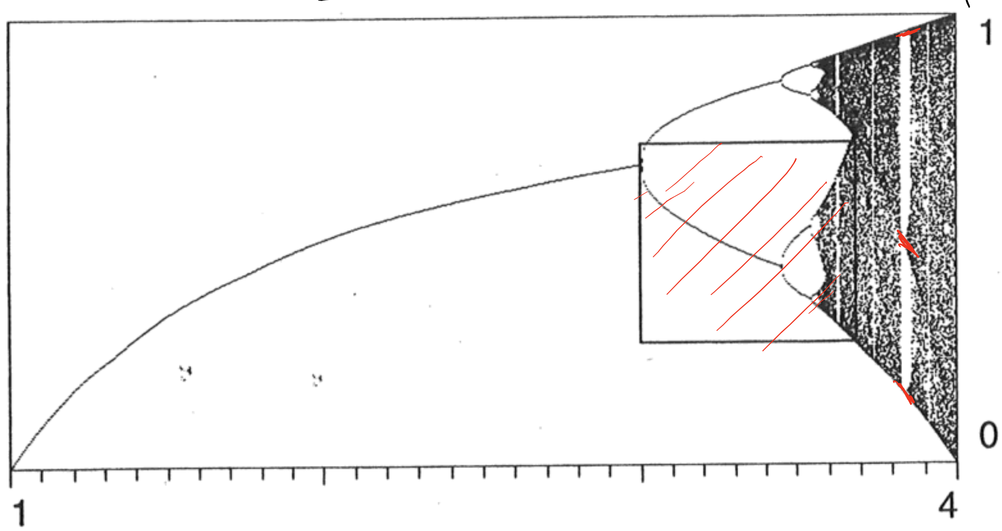




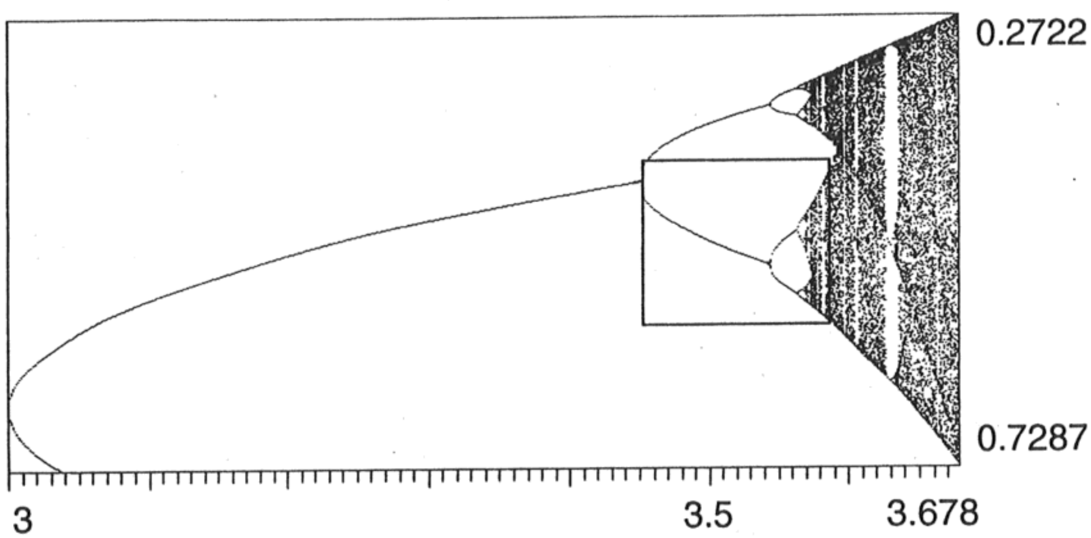




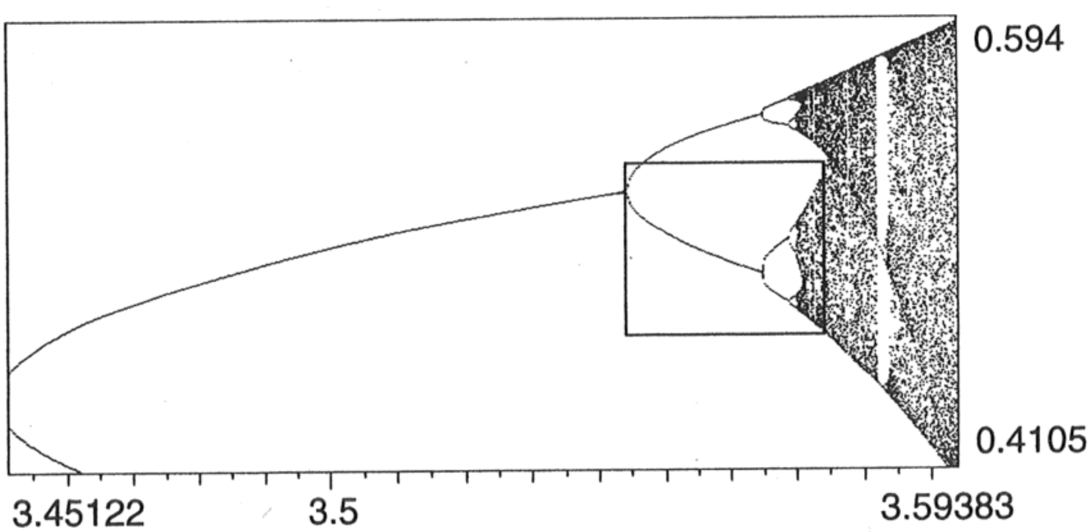
X_∞
0
0
1
2
a
3
4
Feigenbaum Punkt 3.5699456...



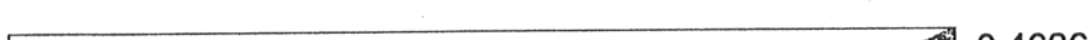
1
0
1
4

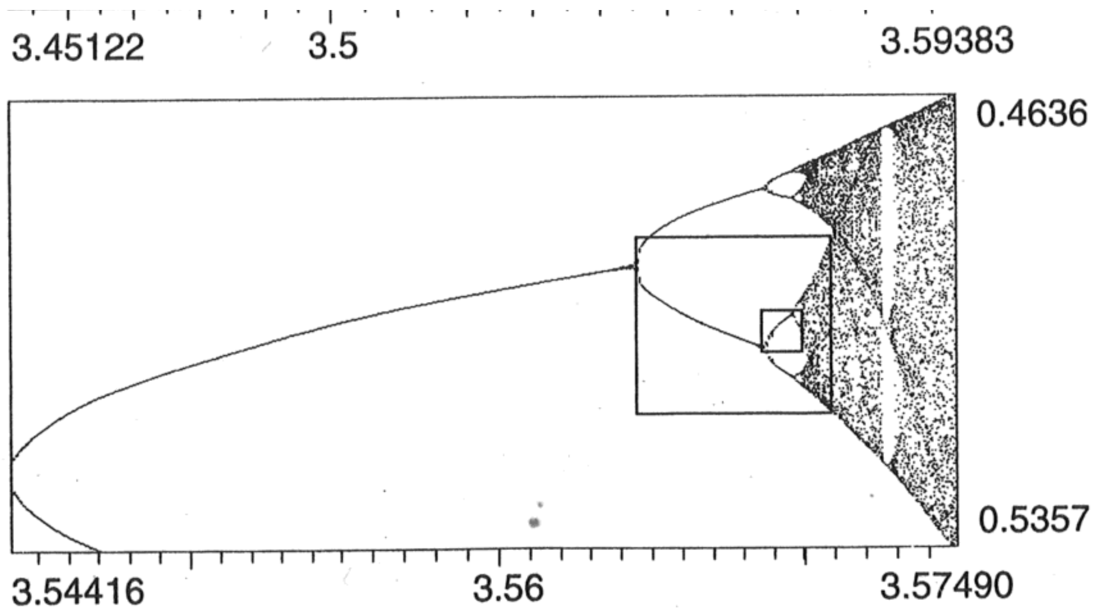


0.2722
0.7287
3
3.5
3.678

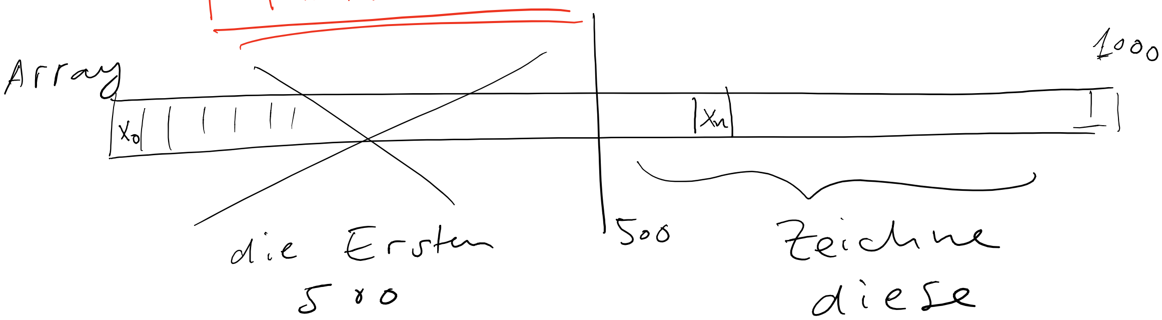


0.594
0.4105
3.45122
3.5
3.59383





FRAKTAL

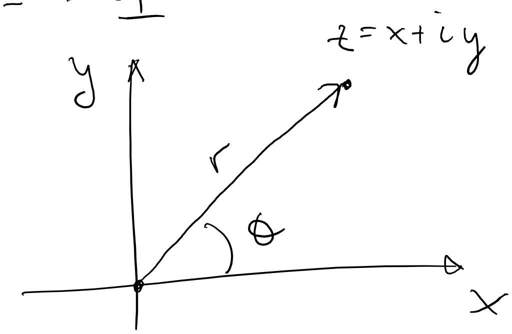


Komplexe Zahlen

$$i^2 = -1$$

$$z = (x + iy)$$

Real



$$z = r e^{i\theta}$$

$$w = (u + iv)$$

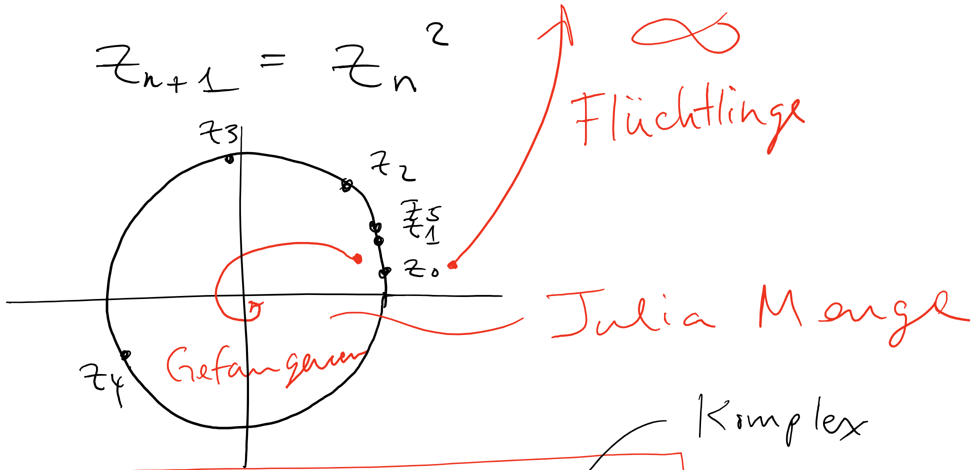
$$z \cdot w = (xu + iyu + ixv + \underbrace{i^2}_{-1} yv)$$

$$= (xu - yv) + i(yu + xv)$$

$$z^2 = (r e^{i\theta})^2 = r^2 e^{i(2\theta)}$$

$$z^2 = \left(re^{i\theta}\right)^2 = r^2 e^{i(2\theta)}$$

$$z_{n+1} = z_n^{\,2}$$



Komplex

$$\boxed{z_{n+1} = z_n^{\,2} + c}$$

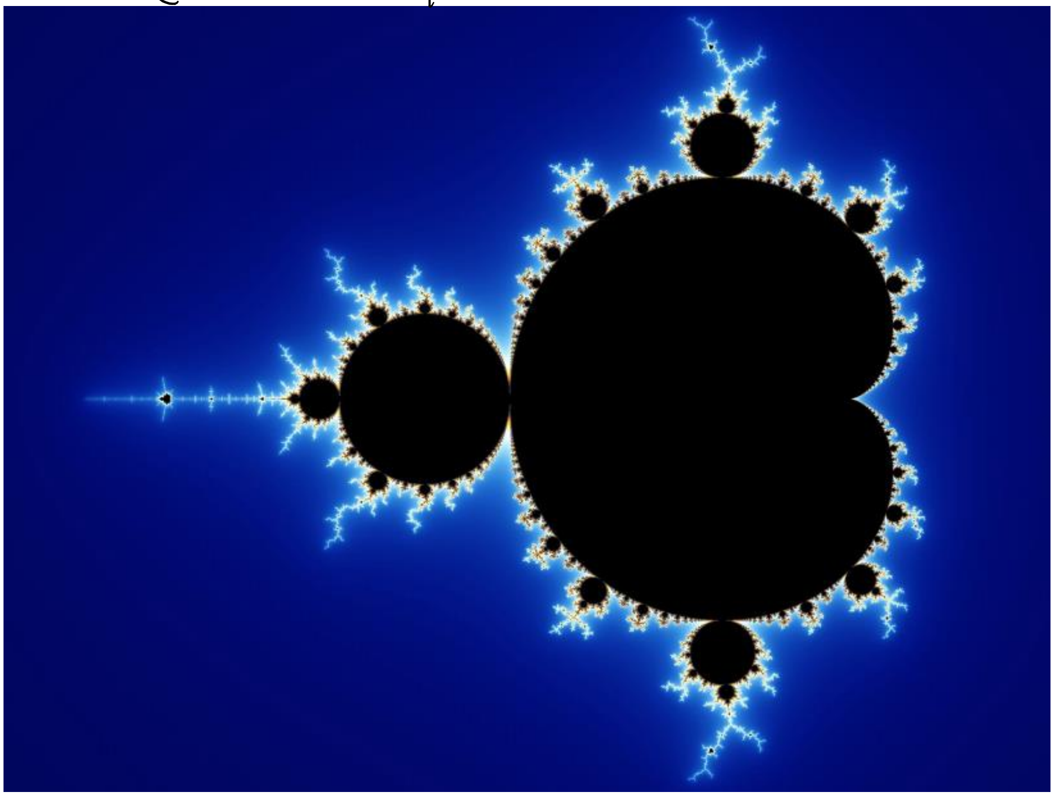
$c = -0{,}5 + 0{,}5\,i$

$$r(c) = \max(|c|, 2)$$

$$M = \{ c \in \mathbb{C} \mid J_c \text{ ist geschlossen} \}$$

$$M = \{ c \in \mathbb{C} \mid c_{n+1} = c_n^{\,2} + c \text{ ist } < \infty \}$$

$M = \{ c \in \mathbb{C} \mid c_{n+1} = c_n^2 + c \text{ ist } < \infty \}$



Für alle c!

```
k = 0
z = c
while (k < 100) {
    if (|z| > 256)
        return (Julia Menge ist Staub)
    z = z*z + c
    k = k+1
}
return (Julia Menge ist geschlossen)
    "Schwarzen Punkt Malen"
```