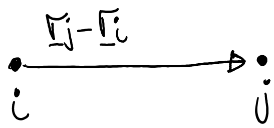
Gravitation und Leapfrog für das Sonnensystem

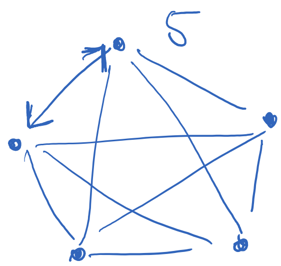
01 April 2019 12:57

8 Planeten & die Sonne



$$\underline{F}_{ij} = \frac{G m_i m_j}{|\underline{r}_j - \underline{r}_i|^2} \frac{(\underline{r}_j - \underline{r}_i)}{|\underline{r}_j - \underline{r}_i|} = \frac{G m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} \underline{r}_j - \underline{r}_i$$

$$\underline{F}_{ji} = -\underline{F}_{ij} \quad \text{(Newton 3)}$$



Wir müssen eigentlich nur 10 Kräfte berechnen.

$$\frac{N(N-1)}{2} \text{ - Kräfte} \quad \mathcal{O}(N^2)$$

$$G_N = 6.6742 \times 10^{-11} \; m^3 \, kg^{-1} s^{-2}$$

$$k = 0.01720209895 \; [AU^{3/2} M_\odot^{-1/2} D^{-1}]$$

$$k^2 = G_N \cdot M_\odot \qquad 1D = 86400 \text{ SI Sekunden}$$

(Tag)

$$\underline{F}_i = \sum_{j \neq i} \frac{k^2 m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} (\underline{r}_j - \underline{r}_i) \longleftarrow \mathcal{O}(N^2)$$

$$\underline{a}_i = \underline{F}_i / m_i \quad \text{(Newton 1)} \longleftarrow \mathcal{O}(N)$$

Leapfrog:

$$\forall i \quad \underline{r}_{1/2,i} = \underline{r}_{0,i} + \frac{h}{2} \underline{v}_{0,i}$$

$$\forall i \quad \underline{v}_{1,i} = \underline{v}_{0,i} + h \, \underline{a}_i(\{\underline{r}_{\frac{1}{2}}\}) \qquad -\nabla\underline{\Phi}$$

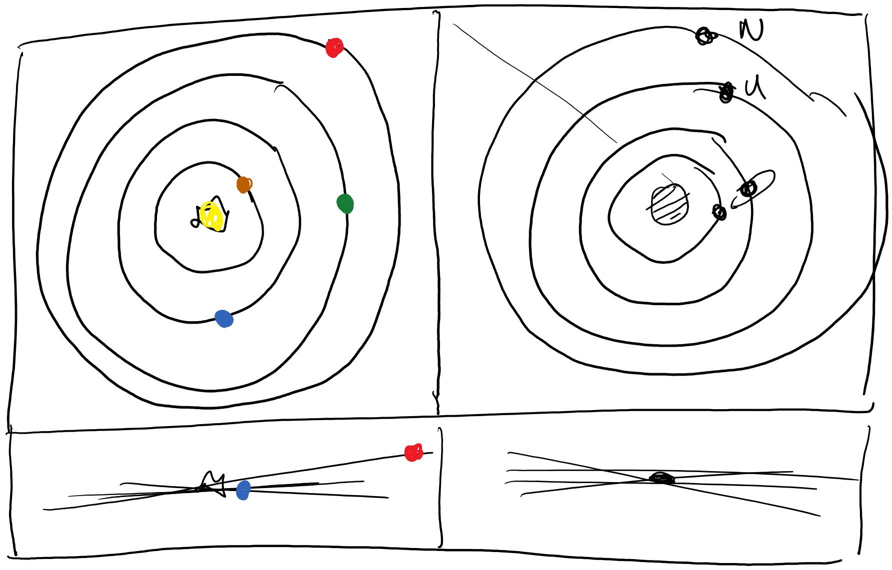
$$\forall i \quad \underline{r}_{1,i} = \underline{r}_{1/2,i} + \frac{h}{2} \underline{v}_{1,i}$$

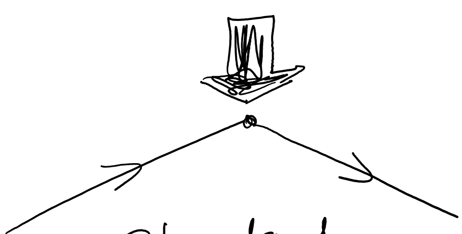
Zeit Schritt h ~ 4 Tage

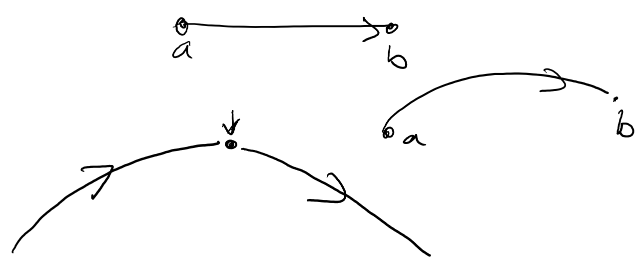
Zeit Schritt h – 4 Tage

File { Anfangsbedingungen : 54 Kordinaten bei Zeit "0"

Massen in Sonnenmassen







Standard Leapfrog

1000 x Genauer!

$$H = T + U$$

$$H = H_{Kepler} + H_{Planet-Planet}$$

State of the Art.

$$\frac{1}{|r|^3} r \longrightarrow r^2 \longrightarrow \left(\frac{1}{\sqrt{r^2}}\right)^3 (\Delta x, \Delta y, \Delta z)$$

$$r^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$