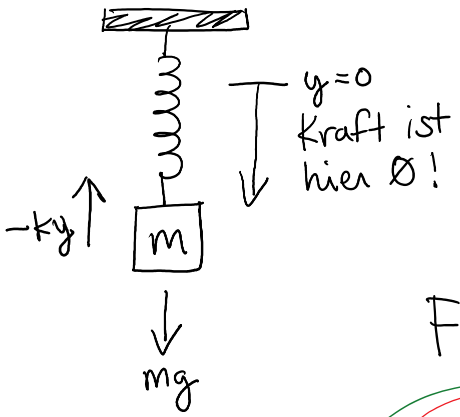
Harmonischer Oscillator



$$F = mg - ky$$

Noch einfacher setzen wir $m=1$ und $k=1$

$$F = g - y$$

Definieren wir q wo $F=0$ ist $q=0$!

$$q = y - g = -F$$

$$\dot{q} = \frac{dq}{dt}$$

Newton: $F = ma = m\ddot{y}$

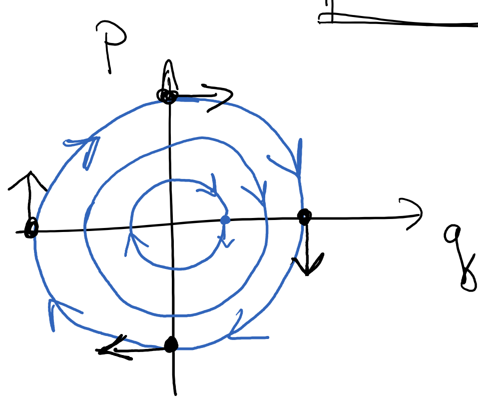
$$\dot{q} = \dot{y} \qquad \ddot{q} = \ddot{y}$$

$$\ddot{q} = \ddot{y} = F = -q$$

Momentum $\underbrace{p = mv = m\dot{y} \overset{m=1}{=} \frac{dq}{dt}}$

$$\boxed{\begin{aligned} \dot{q} &= p \\ \dot{p} &= -q \end{aligned}}$$

2 GDG



$$r^2 = p^2 + q^2$$

Erhalten während der Dynamik

$$H = \frac{1}{2}(\underbrace{p^2}_{\text{Kinetische Energie}} + \underbrace{q^2}_{\text{Potenzielle Energie}})$$

$$\dot{q} = \frac{\partial H}{\partial p}$$

$$\dot{p} = -\frac{\partial H}{\partial q}$$

$$H = T(p) + U(q)$$

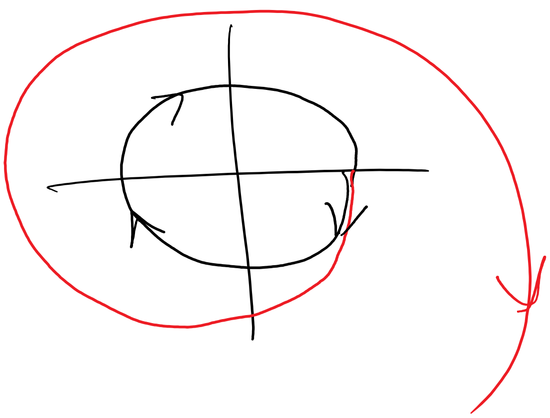
$$\boxed{\dot{p} = -\frac{\partial H}{\partial q}} \qquad H = T(p) + U(q)$$

Vorwärts-Euler Verfahren ?

$$q_{n+1} = q_n + h p_n$$

$$p_{n+1} = p_n - h q_n$$

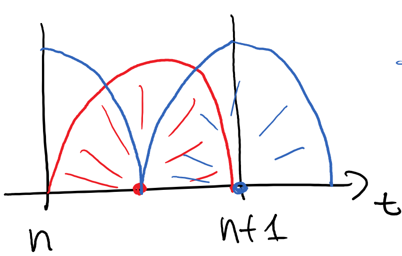
$$q_{n+1}^2 + p_{n+1}^2 = (1 + h^2)(q_n^2 + p_n^2)$$



BAD!

Symplektische Integratoren

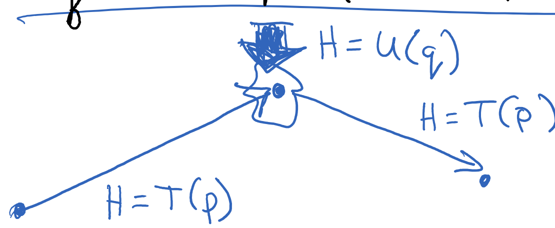
Leap-Frog o. Strömer-Verlet



$$q_{n+\frac{1}{2}} = q_n + \frac{1}{2} h p_n \quad \text{"Drift"}$$

$$p_{n+1} = p_n - h q_{n+\frac{1}{2}} \quad \text{"Kick"} \quad \text{"}\mathcal{O}(h^3)\text{"}$$

$$q_{n+1} = q_{n+\frac{1}{2}} + \frac{1}{2} h p_{n+1} \quad \text{"Drift"}$$

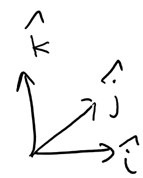


$$\underline{x}_{1/2} = \underline{x}_0 + \frac{1}{2} h \underline{v}_0$$

$$\underline{x}_{1/2} = \underline{x}_0 + \frac{1}{2} h \underline{v}_0$$

$$\underline{v}_1 = \underline{v}_0 + h \underbrace{\left(-\nabla U(\underline{x}_{1/2})\right)}_{\text{Beschleunigung} \equiv F/m}$$

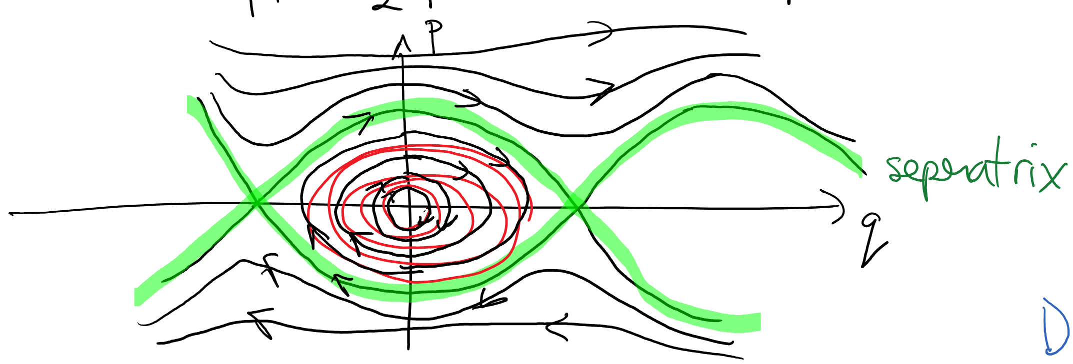
$$\underline{x}_1 = \underline{x}_{\frac{1}{2}} + \frac{1}{2} h \underline{v}_1$$



$$\nabla U = \frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}$$

Einfache Pendel

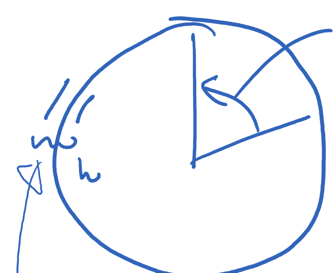
$$H = \frac{1}{2}p^2 - \varepsilon \cos q$$



$H \longmapsto \tilde{H}$

Kontinuierlichen Hamiltonian

Numerischen Hamiltonian ist annähernd zu H mit grösse des Zeitschrittes h.



Der Fehler häuft sich im Winkel auf!

die erhaltene (H) Grösse bleibt immer in einem Band von breite $O(h)$.