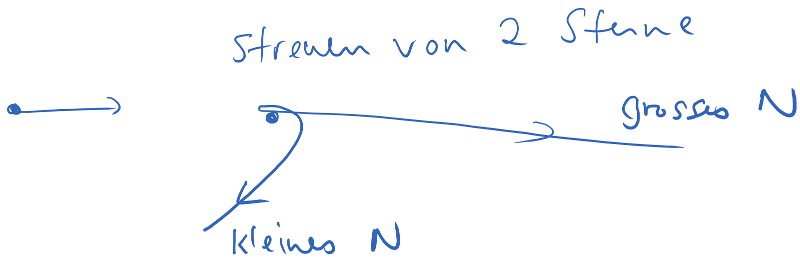
Gravitation Berechnen

Monday, October 24, 2016 1:03 PM

Genauigkeit der Gravitationsberechnung.

$O(N^2)$ Naive

Als N grösser wird:



genaue Kräfte nicht so wichtig wie viele Teilchen (Statistische Genauigkeit)

Wir wollen die Kräfte von a und b vergleichen.

$f_a \gg f_b$ Der relative Fehler

$$\Delta = \frac{\varepsilon_a}{f_a} = \frac{\varepsilon_b}{f_b}$$

Für Maschinengenauigkeit $\sim 10^{-14}$

$$(f \pm [\text{Fehler}]) = (f_a + f_b) \pm \Delta(f_a + f_b)$$

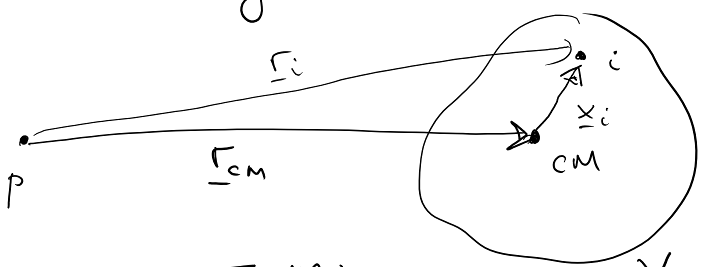
dominiert

Δf_b ist eigentlich viel zu genau für diese Berechnung.

$$= (f_a + f_b) \pm \Delta f_a$$

$$\langle \text{Fehler} \rangle = \sqrt{\Delta^2 f_a^2 + \Delta^2 f_b^2} \quad \text{RMS}$$

Multipole Entwicklung:



$x_i \ll r_{cm}$

$$\psi = - \sum_{i \in V} \frac{m_i}{|\underline{r}_i|} = \sum_{i \in V} m_i \gamma(r_i)$$

$|\underline{r}_i| = r_i \qquad \gamma = -\frac{1}{r}$

$$= \sum_{i \in V} m_i \gamma\left(\left|\underline{r}_{cm} + \underline{x}_i\right|\right)$$

Diapole — Vector index

$$= \sum_{i \in V} m_i \Big[\gamma(r_{cm}) + \frac{\partial}{\partial r^j} \gamma(r_{cm})\, x_i^j$$
$$+ \frac{1}{2} \underbrace{\frac{\partial^2}{\partial r^j \partial r^k} \gamma(r_{cm})}_{\text{2-Rang Tensor}} x^{jk} + \dots \Big]$$

Teilchen index

Quadrupole

Wie die Ableitungen der Green's Function zu berechnen?

$$\gamma(r) \equiv \gamma_0 = -\frac{1}{r} \quad ; \quad \gamma_{m+1} = -\frac{(2m+1)}{r^2} \gamma_m$$

$$\frac{\partial}{\partial r} \gamma_m = \gamma_{m+1} \underline{r}$$

$$\partial \gamma_0 = -\frac{1}{r^2} \gamma_0 \underline{r} = \frac{\underline{r}}{r^3} \qquad r \equiv r_{cm}$$

$$= \sum_{i\in V} m_i \left[-\frac{1}{r} + \frac{1}{r^3} r_j x^j + \partial\partial\gamma(r_{cm}) \right]$$

$$``\underline{r}\cdot\underline{x}"$$

$$\partial(\gamma_{\underline{1}}\, \underline{r}) = \gamma_2\, \underline{r}\,\underline{r} + \gamma_{\underline{1}}\, \delta$$

$$\gamma_2 r_{jk} + \gamma_{\underline{1}} \delta_{jk}$$

$$\gamma = -\frac{3}{r^2}\gamma_{\underline{1}} = -\frac{3}{r^5}$$

$$\Psi = \sum_{i\in V} m_i \left[-\frac{1}{r} + \frac{1}{r^3} r_j x_i^j + \frac{1}{r^3}\delta_{jk} x_i^{jk} - \frac{3}{r^5} r_{jk} x^{jk} \right]$$

$$M = \sum_{i\in V} m_i \qquad M^j = \sum m_i x_i^j \overset{!}{=} \emptyset$$

die Definition des Massenzent.

$$M^{jk} = \sum_{i\in V} m_i x_i^{jk}$$

Quadrupole Moment

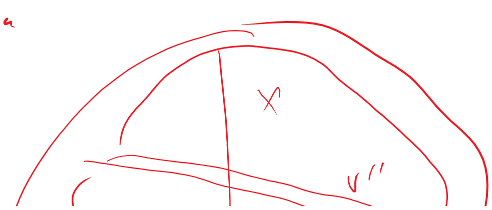
$$\Psi = -\frac{M}{r} + \cancel{\frac{r_j}{r^3} M^j} + \frac{1}{r^3}\mathrm{Tr}(M^{jk}) - \frac{3}{r^5} r_{jk} M^{jk}$$

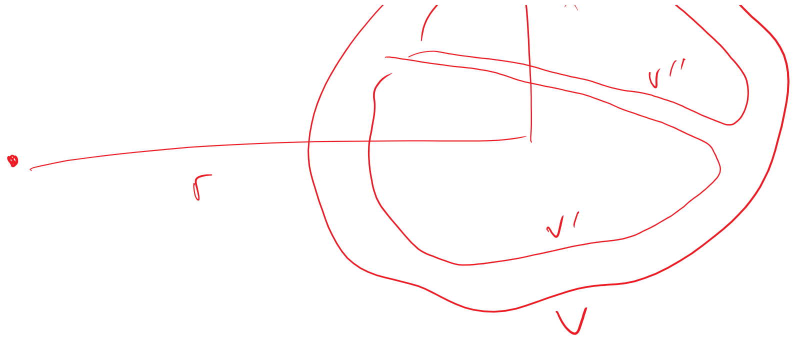
+ Fehler

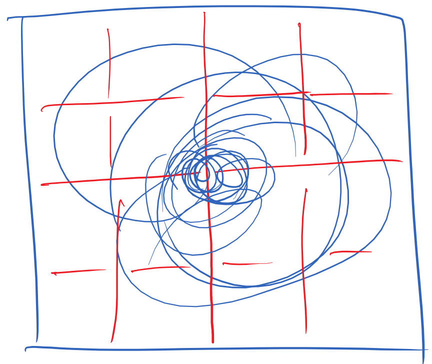
$$\partial^l \Psi \equiv a^l$$

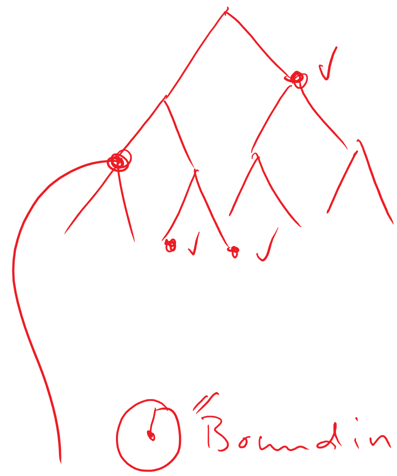
Beschleunigung $(\underline{F} = m\underline{a})$

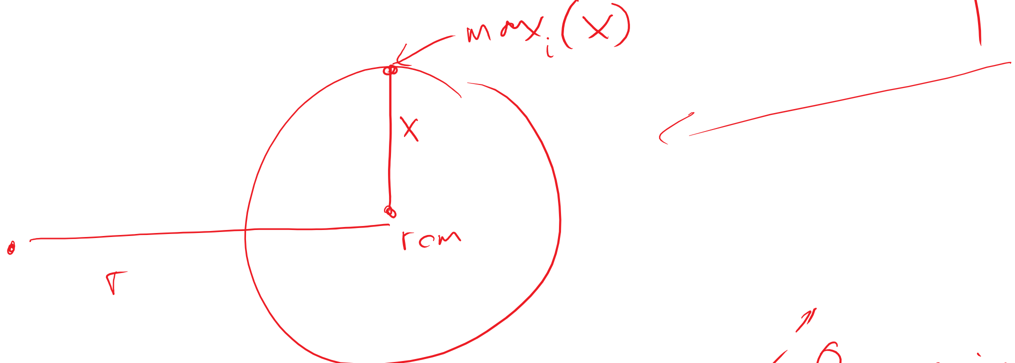
Wie steuere ich die gewünschte Genauigkeit?











⊙ "Bounding"

M

$M^{jk} \leftarrow 6$

r_{cm}^{j}

"Opening Angle"

$$\max(x) < \Theta \cdot r$$

$$O(N \log N)$$

$$\Theta \sim 0.6$$

RMS Relativer Fehler

von $\sim 0.1\%$