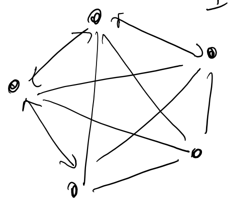
The Rise of Trees

Monday, September 18, 2017 1:08 PM

5 planets
10 interactions



$O(N^2)$

$$N_{inter} = \frac{N(N-1)}{2} \quad (F_{ij} = -F_{ji})$$

$N = 2 \times 10^{12}$ Particles

$O(N^2)$ $= 2 \times 10^{24}$ Interactions

20 Flops / Interaction

$\Rightarrow 4 \times 10^{25}$ Flops

Petaflop Computing $\longrightarrow 10^{15}/s$

Forces Time $\longrightarrow 4 \times 10^{10} s$

$\sim 10^3$ Years

$O(N)$ $O(N \log N)$

Spectral Method.

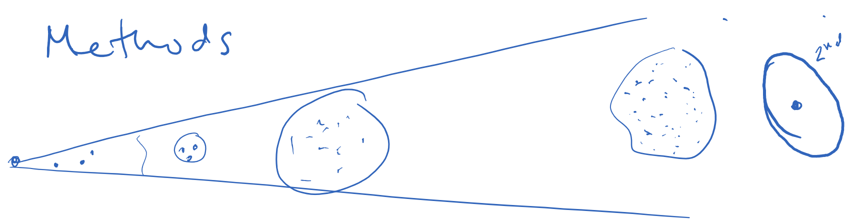
$\nabla^2 \phi = \rho(\underline{r})$ FFT $(O(N \log N))$

$-k^2 \phi_k = \rho_{\underline{k}}$

$\phi(\underline{r})$ IFFT

Particle - Mesh

Multipole Methods

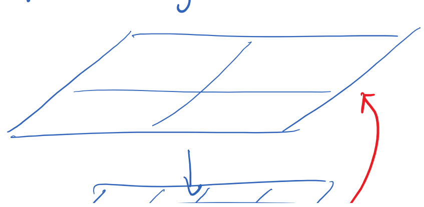


Trees

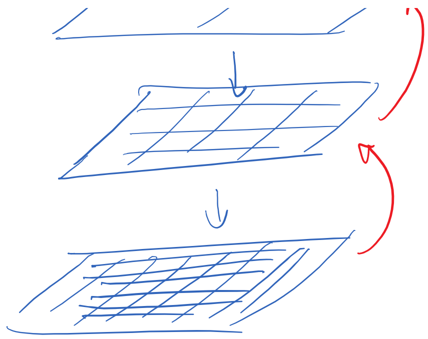
"Treecode" $\longrightarrow O(N \log N)$

Fast Multipole Method $O(N)$

Multigrid $\longrightarrow$ S.O.R.

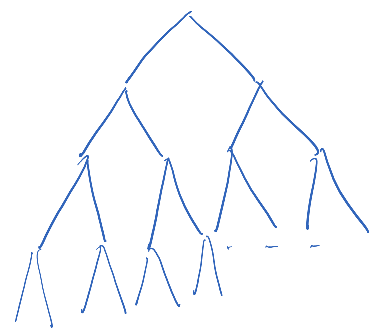


$O(N)$ Method.

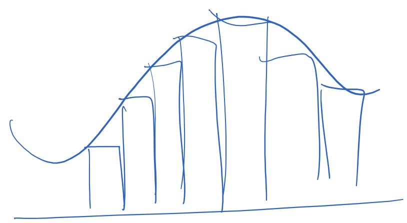


O(N) Method.

$$\frac{\text{Flop}}{\text{Byte}} > 10 \sim \text{low}$$



O(N log N)
"Sort"
quicksort

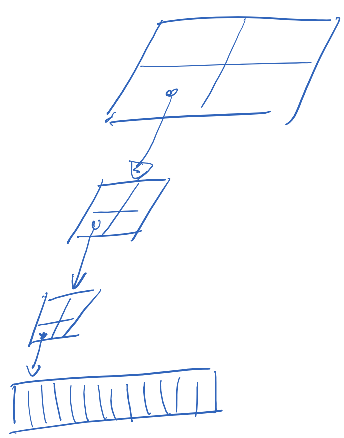


Radix sort.

Sorting of keys

Sort is O(N) if the keys are drawn from a distribution which is Rieman Integrable.

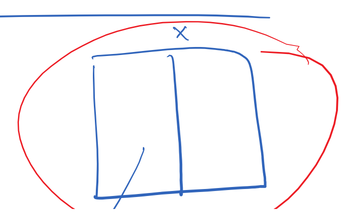
2-D ?

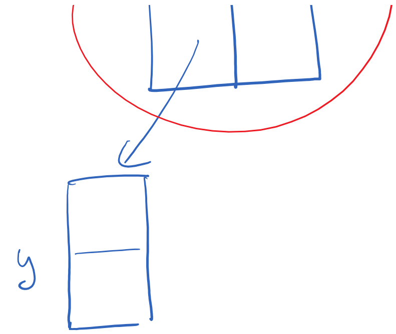


Quad tree

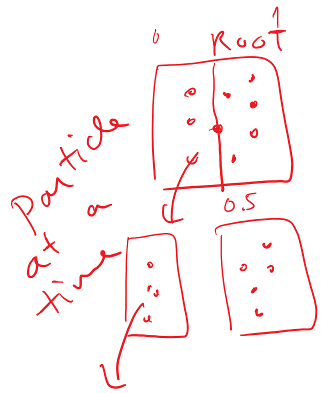
3-D Oct-tree

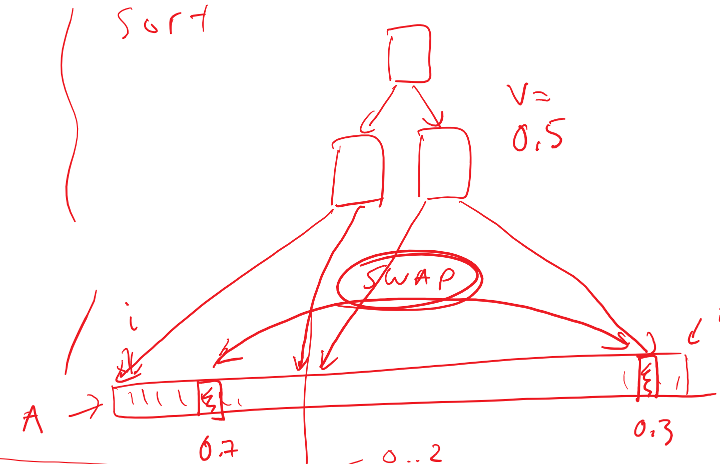
Binary





~~K-D Tree~~ Gravity





S = partition(A, i, j, V, d)

Index of the first element between i and j (inclusive) for which A.r[d] $\geq$ V.

☆ 1. Most Compact Code

2. Fastest Code
- Count SWAPs
- Count $\leq$