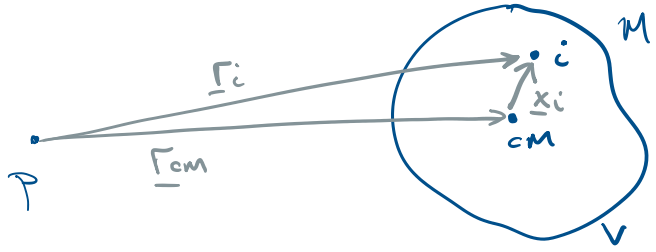
Gravity Tree Walk

16 October 2017 15:06



$|x_i| \ll |r_{cm}|$

$$\Psi = -\sum_{i\in V} \frac{m_i}{|r_i|} = \sum_{i\in V} m_i \gamma(r_i) \qquad \gamma = -\frac{1}{r}$$

$$= \sum_{i\in V} m_i \gamma\left(|r_{cm} + x_i|\right)$$

$$= \sum_{i\in V} m_i \left[\gamma(r_{cm}) + \frac{\partial}{\partial r^j}\gamma(r_{cm}) x_i^j + \frac{1}{2}\frac{\partial^2}{\partial r^j \partial r^k}\gamma(r_{cm}) x_i^{jk} + \dots \right]$$

Dipole, vector index

$x^j x^k \equiv$ [xx xy xz; . yy yz; . . zz], i

∂_{jk} — Rank 2 Tensor — Quadrupole

What about the derivatives of the Green's function?

$$\gamma(r) \equiv \gamma_0 = -\frac{1}{r}; \qquad \gamma_{m+1} = -\frac{(2m+1)}{r^2}\gamma_m$$

$$\frac{\partial}{\partial r}\gamma_m = \gamma_{m+1} r$$

$$\partial\gamma_0 = -\frac{1}{r^2}\gamma_0 \underline{r} = \frac{\underline{r}}{r^3} \qquad \underline{r} \equiv r_{cm}!$$

$$= \sum_{i\in V} m_i \left[-\frac{1}{r} + \frac{1}{r^3} r_j x_i^j + \frac{1}{r^3}\delta_{jk} x_i^{jk} - \frac{3}{r^5} r_{jk} x_i^{jk} \right]$$

$$\partial(\gamma_1 \underline{r}) = \gamma_2 \underline{r}\,\underline{r} + \gamma_1 \delta = \gamma_2 r_{jk} + \gamma_1 \delta_{jk}$$

$$\gamma_2 = -\frac{3}{r^2}\gamma_1 = -\frac{3}{r^5}$$

$$M = \sum_{i\in V} m_i \qquad M^j = \sum_{i\in V} m_i x_i^j \qquad M^{jk} = \sum_{i\in V} m_i x^{jk}$$

$$M = \sum_{i \in V} m_i \qquad \left(M^j = \sum_{i \in V} m_i x_i^j\right) \quad M^j = \sum_{i \in V} m_i x^j$$

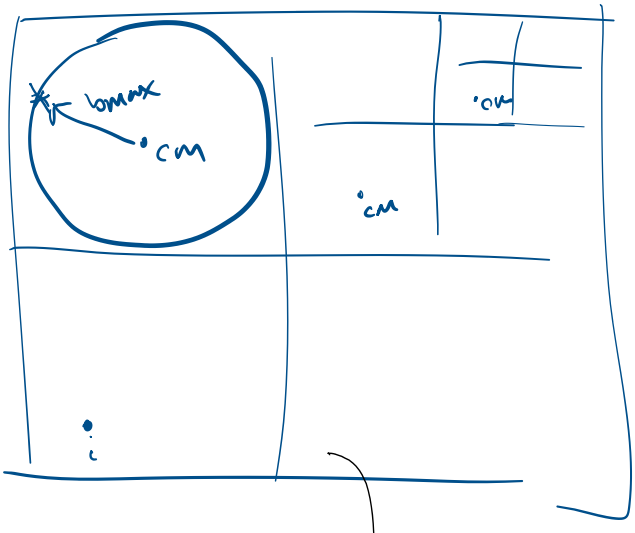
$= 0$! if we expand about the center of Mass.

Mass

$$\psi = -\frac{M}{r} + \frac{1}{r^3} T_r(M^{jk}) - \frac{3}{r^5} r_{jk} M^{jk} + \cdots$$

? $-\partial^e \psi \equiv a^e$

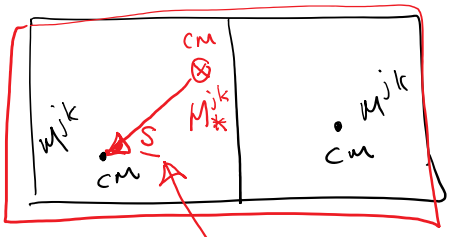
↳ Acceleration $\quad \underline{F} = m\underline{a}$



Cell Structure should now contain:

$\underline{r}_{cm}$, M, M^{jk}, $\tilde{b}_{max}$

6 components

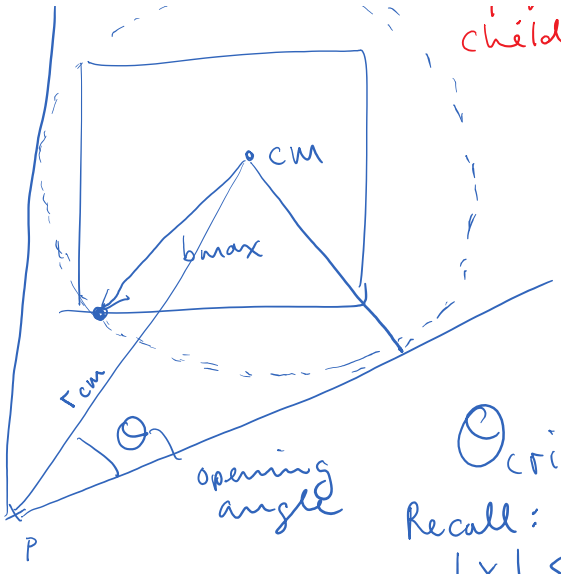


How to shift a moment:

$$M^{jk}_* = M s^{jk} + M^{jk}$$

vector s! points from new center to the center of mass of the child cell.

child cell.



$\theta_{crit} \approx 0.7$

Recall: $|x| \ll |r_{cm}|$ in the expansion!

$\theta < \theta_{crit}$

if $\left(\frac{b_{max}}{r_{cm}} < \theta_{crit}\right)$ then accept Multipole expansion

else open this cell.