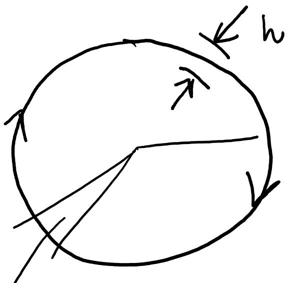
Gravitation for many bodies: a digital Orrery

26 October 2020 12:51



$\delta\theta$ Phase error improves with decreasing h!

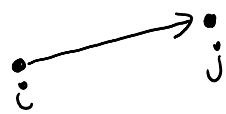
8 Planets + Sun

9 bodies

$$H = T + U$$

Kinetic Energy

Potential Energy (9 bodies)

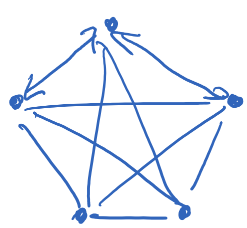


$$\underline{F}_{ij} = \frac{\boxed{G} m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} (\underline{r}_j - \underline{r}_i) \qquad \boxed{i \neq j}$$

Newton's Law of Gravity.

$$\boxed{\underline{F}_{ij} = - \underline{F}_{ji}}$$

Newton's 3rd law



$5 \times 4 = \frac{20}{2} = 10$ interactions

20 Forces

N-bodies:

$$\frac{N(N-1)}{2} \quad O(N^2) \text{ interactions to calculate}$$

✓ $O(N \log N)$

✓ $O(N)$ 10 trillions $\sim 10^{13}$

$$G_N = 6.6742 \times 10^{-11} \, [m^3 kg^{-1} s^{-1}]$$

$$G_N \cdot M_\odot = k^2 \quad \text{Gauss' Grav. Const.}$$

$$k = 0.01720209895 \, [AU^{3/2} M_\odot^{-1/2} D^{-1}]$$

$$\boxed{\underline{F}_i = \sum_{i \neq i} \frac{k^2 m_i m_j}{|\underline{r}_j - \underline{r}_i|^3} (\underline{r}_j - \underline{r}_i)}$$

$$\underline{F}_i = \sum_{j\neq i} \frac{\quad}{|\underline{r}_j - \underline{r}_i|^3}(\underline{r}_j - \underline{r}_i)$$

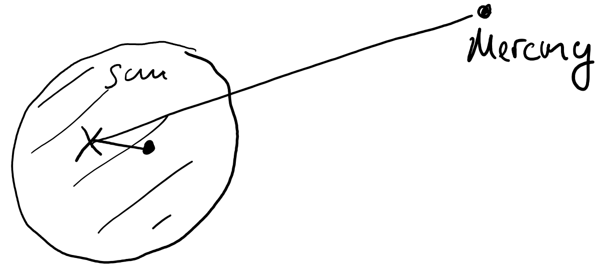
$$\underline{a}_i = \ddot{\underline{r}}_i = \frac{d^2\underline{r}_i}{dt^2}$$

$$\underline{a}_i = -\nabla_i U$$

$1D = 86400$ S.I. seconds.

$$\underline{a}_i = \frac{\underline{F}_i}{m_i}$$

equation of Motion



Leapfrog

$O(N)$ $\underline{r}_{\frac{1}{2},i} = \underline{r}_{0,i} + \frac{h}{2}\underline{v}_{0,i}$ "H=T"

$O(N^2)$ $\underline{v}_{1,i} = \underline{v}_{0,i} + h\,\underline{a}_i(\{\underline{r}_{\frac{1}{2}}\})$ "H=U" Code below

$O(N)$ $\underline{r}_{1,i} = \underline{r}_{\frac{1}{2},i} + \frac{h}{2}\underline{v}_{1,i}$ "H=T"

$$T = \sum_{j=0}^{8} \underbrace{m_j \underline{v}_j^2}_{T_j}$$

"Drift"

$$\dot{q} = \frac{\partial H}{\partial p}$$

$$\dot{\underline{r}}_i = \frac{\partial \overset{T}{H}}{\partial \underline{p}_i} = \frac{\partial T}{\partial \underline{p}_i}$$

$$\dot{\underline{r}}_i = \frac{\partial T_i}{\partial \underline{p}_i} = \underline{v}_i$$

1. code

$\underline{\Delta r} = \underline{r}_j - \underline{r}_i$ 3⊖ ⊜

r2 = Δx * Δx + Δy * Δy + Δz * Δz 3⊛ 2⊕ 1⊜

ir = $1/\sqrt{r2}$ 1⊛ ⊜

ir3 = ir * ir * ir 2⊛ 1⊜ $\left(\frac{1}{|r_j - r_i|^3}\right)$

mir3 = mi * mj * ir3

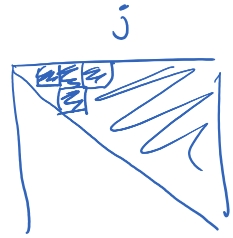
fx = mir3 * Δx

fy = mir3 * Δy

$f_z = mir3 * \Delta z$

$\underline{a}[i] \mathrel{+}= \underline{f} * \underbrace{im[i]}_{\frac{1}{m_i}}$

$\underline{a}[j] \mathrel{-}= f * im[j]$

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for (i=0; i<N; ++i) {
    for (j=i+1; j<N; ++j) {
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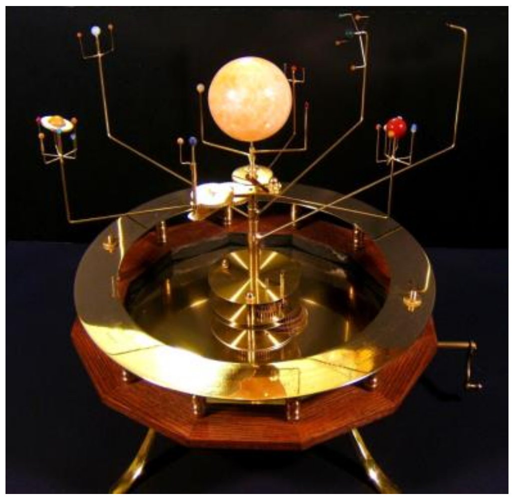
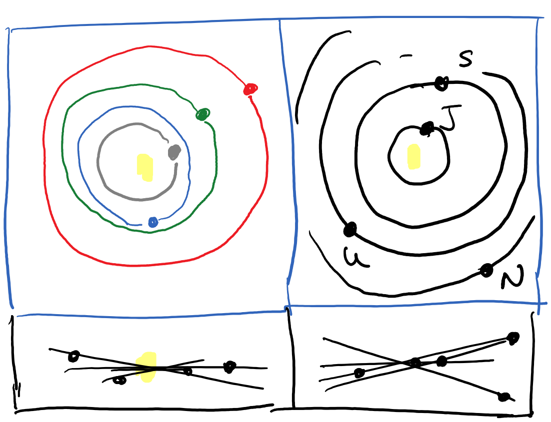
$\Theta(N^2) \longrightarrow$ ////// Code from above

3 }

3 }

$\boxed{\text{We need I.C.s in A.U. } \underline{r} \quad \text{A.U./Day } \underline{v} \quad m \text{ in } M_\odot}$





$\Delta t = 4$ days

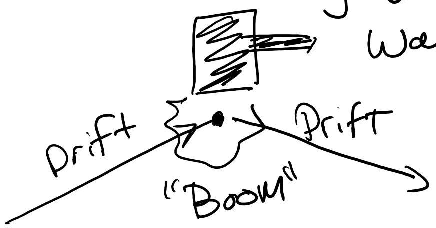
$$H = T(p) + U(q)$$

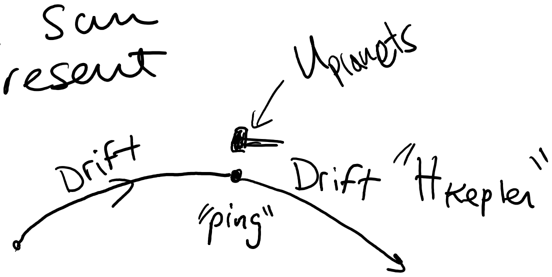
$$H = H_{KEPLER}(p) + U(q)$$

includes the Sun

Potential of all the Planets only 1000 weaker

Free particle is motion as if just the Sun was present





State-of-the-art

1000x Better!

Still $O(N^2)$